Crash course – Higgs Boson decays

Quantum Field Theory, 1SV037, spring 2011

Uppsala University - Department of Physics and Astronomy

Date and location: 19 May, 13-17, room 6K1113.

24 May, 10-12, room 80109 and 15-17 room 12167

Instructor: Glenn Wouda

E-mail: `glenn.wouda@fysast.uu.se`

www: `http://www.isv.uu.se/~wouda/`

Office: 12142

Phone: 018-471 38 55

1 Introduction

The Large Hadron Collider (LHC) at CERN has finally started its quest for finding the last missing piece of the Standard Model (SM), the *Higgs boson.* Without the Higgs boson in the SM, the fermions (leptons and quarks) and the exchange bosons of the weak interaction ($Z^0, W^{\pm}$) should be massless. Despite the fact that the Higgs boson was postulated already in the mid 1960's – it has not been found experimentally yet. One of the reasons for why that is so is that its mass is not predicted by theory – it is a free parameter. However, the mass can not attain arbitrary values. If the Higgs boson exists, it will influence the values of other measurable quantities in the SM through quantum loops (*e.g.* the anomalous magnetic moment of the muon, Peskin problem 6.3). Precision measurement of such parameters[1] can constrain the Higgs boson mass to $\lesssim$ 200 GeV.

It is important to know how the Higgs boson will decay depending on what mass it has so that one know what to look for in the detector. In this exercise, we will calculate the

[1] Called Electroweak precision Tests (EWPT)

decay widths and branching ratios of the Higgs boson into fermions and massive vector bosons starting from the SM Lagrangian.

This **exercise** will be performed as interactive problem solving sessions. Interactive means that the instructor will lead discussion and calculations on the blackboard together with the students. It is important to read the instructions before the exercise takes place to get the most out of it.

The **examination** is to answer all the questions in this exercise and presenting the results of the calculation of the Higgs boson decay width and branching ratios in a short written report (handwritten are OK, except the figures which you need a computer for).

Deadline: 3 June

Reference: Peskin and Schroeder, chapters 4, 5, 11, 15, 20.

2 The Electroweak Standard Model

The aim here is just give a brief background to why we are concerned with the Higgs boson at all. In order to perform the problems in this exercise only knowledge from the QFT course (Peskin part 1) is required.

2.1 Gauge Invariance

The Standard Model of electroweak interactions is a so called *gauge theory*, this means that the fields are subject to <u>local</u> symmetry transformations, *i.e.* the transformation depends on the space-time point. This is to be compared with <u>global</u> symmetries where we had Noether's Theorem and all that. The beauty of gauge theory is that it unites forces (exchange bosons) with symmetry. Many of the topics involved in gauge-theory, such as Path Integral quantization, gauge fixing, ghosts, spontaneous symmetry breaking etc., will be covered in more advanced QFT and Particle Physics courses [2]. The aim of

[2] Peskin part 2 and 3 basically.

this exercise is to give you a small flavour of some of these ideas and to produce results which are actually being used in contemporary particle physics research!

Quantum Electrodynamics (QED) can be derived from the postulate that the Lagrangian governing electromagnetism is invariant under local phase transformations, in a fancy wording: QED is a U(1) Gauge Theory! One very important concept in modern physics is the unification of forces. The unification of electromagmetism and the weak force (responsible for beta decay of atomic nuclei) is called the Electroweak force. The symmetry behind the Electroweak force is SU(2)×U(1) where the SU(2) symmetry only acts on the left-handed fermion fields! The reason for that is that one observed that left- and right handed fermions interacts differently with the W boson.

In gauge theory, there is one exchange boson for each symmetry generator, thus in SU(2)×U(1) we have $3 + 1 = 4$ such. At the time of the construction of the Electroweak theory, one knew of the charged weak current. In this way, one "knew" about the W bosons as well. So any theory that would give us electroweak unification should give us at least three exchange bosons, the W's and the γ. When the Z boson was discovered, it was clear that SU(2)×U(1) was the correct choice since one also got the correct mass for the Z with this symmetry group!

Now, in gauge theory, mass terms for the exchange bosons are forbidden due to gauge invariance, hence the force carriers are massless if we assume local symmetry. This is indeed true for QED and Quantum Chromodynamics (QCD) since the photon and gluons are massless. But we know already from the start that the force carriers of the weak force is massive, that is why that force is weak! It is a very short ranged force due to the massive nature of it's exchange particles.

We need one more concept to realize both gauge invariance and massive bosons, the solution is called *the Higgs Mechanism.* This mechanism postulates a new field, the *Higgs field* which has a non-zero vacuum. The more technical name is *spontaneous symmetry breaking* which means that the Lagrangian and hence the dynamics has all the imposed symmetries whereas the ground state does NOT have the full symmetry. We can accommodate masses to the force carriers while maintaining gauge invariance! The particles interacting with the vacuum of this field will thus experience some inertia which will be realized as mass for those particles. Remember that in Quantum Field Theory, the particles are field excitations and the vacuum is then the absence of particles. The Higgs field will thus influence the other particles even in the absence of excitations of it!

Since we break the Left-Right symmetry of the fermions, this means that we can not have fermion masses in the standard model. Fortunately if we couple the fermions to the Higgs field through Yukawa couplings, we can give them mass. We include the yukawa couplings in the theory since they are allowed by symmetry and everything that is allowed can/will occur in Quantum theory.

The strength of this Electroweak theory is that it predicts one new boson which was found at CERN in 1983 with the correct mass predicted by the theory! There is however one missing piece, the excitations of the Higgs Field – the *Higgs Boson*. This particle has not been found yet and its mass is a free parameter of the Standard Model.

Problem 1

Consider the free Dirac Lagrangian $\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$. Discuss the outcome of a chiral transformation of this lagrangian, i.e. that left- and right handed fields transforms differently under a certain symmetry. How is this related to the standard model?
Hint: consinder global transformations.

2.2 The Lagrangian

In this section we will examine the relevant parts of the Lagrangian for decays of the Higgs boson. The Higgs field interacts with the exchange bosons through the so called *covariant derivative*:

$$D_\mu = \partial_\mu - ig_2\tfrac{1}{2}B^a_\mu\sigma^a - ig_1\tfrac{1}{2}C_\mu$$

This is the essential part of all gauge theories, that we replace the ordinary derivative with this expression in order to maintain invariance under local transformations. We do so by including interactions with new fields, one for each generator of the symmetry. The second term on the right hand side is the SU(2) part, the index a is to be summed over and runs from 1 to 3 (since SU(2) has three generators – σ^a, the Pauli matrices), the B^a_μ are then the corresponding fields and g_2 is a *coupling constant* which determines the strength of the interaction. The third term is the U(1) part of the standard model symmetry and has only one field since U(1) can be described by just one parameter. The factors one half is just a conventional normalization.

For QED the covariant derivative is:

$$D_\mu{}^{QED} = \partial_\mu - ieA_\mu$$

where A_μ is the photon field and the coupling constant is just the electric charge!

The kinetic term for the Higgs field $|\partial_\mu \Phi|^2$ is thus replaced by:

$$|D_\mu \Phi|^2 \, .$$

The Higgs field is a complex scalar field which is a SU(2) doublet, i.e. it transforms under the fundamental representation of SU(2) and is written as:

$$\Phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

We now postulate that by expanding around the field minima, we obtain:

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

where v is the minimum different from zero and $h(x)$ is the excitation about this minima and is then the Higgs boson. As can be seen by inspection it was the ϕ_3 field that had a non zero minima. The other three fields has zero vacuum eenrgy and the excitations of those are called *goldstone bosons* and can be set to zero due to the gauge invariance, we can transform the Higgs field so these bosons disappear from the spectrum.

By inserting the Higgs field into the kinetic term, we will generate masses for the boson fields B_μ^a and C_μ. The the boson fields appearing in the covariant derivative are not the physical (mass) eigenstates but are found (as in normal QM) by diagonalizing the Hamiltonian (in this case the Lagrangian). One first has the charged W bosons: $W_\mu^\pm = (B_\mu^1 \mp iB_\mu^2)/\sqrt{2}$ which gains a mass of $g_2 v/2$. After that there is one massless particle, the photon: $A_\mu = (g_1 B_\mu^3 + g_2 C_\mu)/\sqrt{g_1^2 + g_2^2}$ and the massive Z boson $Z_\mu = (g_2 B_\mu^3 - g_1 C_\mu)/\sqrt{g_1^2 + g_2^2}$ which has the mass of $\sqrt{g_2^2 + g_1^2}\, v/2$.

Problem 2

By inserting the Higgs field into the kinetic term and using the physical states and masses for the exchange bosons, find the Feynman rules for Higgs decays into these exchange bosons. Can the Higgs boson decay into two photons? If it can't, can you think of any reasons for why that is so? What is the similarity and difference between the hZZ and hW^+W^- vertices?

3 Massive Vector Bosons

Vector bosons are as the name suggests described by vector fields, which transforms under Lorentz transformations as

$$V^\mu(x) \to (\Lambda^{-1})^\mu{}_\nu V^\nu(\Lambda \cdot x).$$

Recall that the wave-equation for a massless vector boson, such as the photon, came from the Lagrangian:

$$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \to \partial_\mu F^{\mu\nu} = 0,$$

where we also have to specify one futher condition (the Lorentz condition $\partial_\mu V^\mu = 0$) to reduce the number of degrees of freedom from four to three:

$$\partial^2 V^\mu = 0,$$

(Peskin eq. 4.130) where V^μ is the (massless) vector field. For massive vector fields, we have the Lagrangian:

$$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}M^2 V^\mu V_\mu, \tag{1}$$

where the field V^μ is assumed to be real, for a complex field we have M^2 instead of $M^2/2$. The wave equation that we get from this Lagrangian is called the *Proca equation* and looks like:

$$\tfrac{1}{2}(\partial^2 + M^2)V^\mu = 0 \tag{2}$$

The general, plane wave, solution for the vector boson is, Peskin (4.131):

$$V^\mu(x) = \sum_\lambda \int \frac{d^3p}{\sqrt{2E}(2\pi)^3} \left[a_\lambda(p)\,\epsilon^\mu(p,\lambda)\,e^{-ipx} + a^*_\lambda(p)\,\epsilon^{\mu*}(p,\lambda)\,e^{ipx}\right] \tag{3}$$

since the boson is now massive, there is a frame in which it is at rest, thus we have three spin polarizations, λ. The three ϵ^μ's are called *polarization four-vectors*, they describe the directions in Minkowski space for each polarization and are linearly independent. Compare this with the plane wave solution for the Dirac field, where the spinors u and v encodes the spin nature of the field. In same way, the polarization four vectors contains the (Lorentz) vector nature of the vector field.

The polarization vectors are lineary independent and form a complete basis:

$$\epsilon^{\mu *}(p,\lambda)\epsilon_\mu(p,\lambda') = -\delta_{\lambda\,\lambda'} \tag{4}$$

and the also obey the Lorentz condition:

$$\partial_\mu V^\mu = 0 \Rightarrow p^\mu \epsilon_\mu(p,\lambda) = 0$$

For the photon, there is no frame of reference such that is it at rest, thus we only have 2 polarization four-vectors for photons. The photon does not have a longitudinal polarization, see Peskin page 124. Now the massive vectors have such frame of reference, where it has 3 different axes of rotation, we can choose them to be: $\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$, so that in the rest frame the polarization four vector becomes: $\epsilon^0(0,\lambda) = 0,\ \forall\lambda$, and $\epsilon^i(0,\lambda) = \mathbf{e}_\lambda,\ \ \lambda = 1(x), 2(y), 3(z)$. thus:

$$\epsilon^\mu(0,\lambda) = (\epsilon^0(0,\lambda), \vec{\epsilon}(0,\lambda)).$$

Now assume that the vector boson is moving in the z-direction, $p^\mu = (E, 0, 0, p_z)$, then by boosting the polarization four vectors into this frame, we see that we must have

$$\epsilon^\mu(p,3) = (p_z, 0, 0, E)/M,$$

in order to fulfill the Lorentz condition: $p^\mu\epsilon_\mu(p,3) = 0$. Also notice that in the limit $p_z = 0$ we get the correct form for the polarization four vectors in the rest frame.

The polarization four vectors for massive particles form a completeness relation.

$$\sum_\lambda \epsilon^{\mu *}(p,\lambda)\epsilon^\nu(p,\lambda) = -g^{\mu\nu} + \frac{p^\mu p^\nu}{M^2} \tag{5}$$

Which is to be compared with the case for massless vectors, Peskin (5.75). The Feynman Rules for external massive vector bosons works in the same way as for photons in chapter 4.8 of Peskin but with the completeness relation (5) instead.

Problem 3

Calculate the partial decay width for the Higgs boson decaying into a pair of W bosons and Z bosons, use the Feynman rules derived in problem 2 and equation (4.86) in Peskin. You will need the 2-body phase space (4.83), what is E_{cm} in this case? What is the behaviour for $m_h >> m_W, m_Z$?

4 Higgs couplings to fermions

The Higgs field Φ can and will couple to fermions through Yukawa couplings:

$$-y_f\,\bar{\Psi}_f(x)\Psi_f(x)\phi_3 \to -y_f\,\bar{\Psi}_f(x)\Psi_f(x)(v+h(x)), \tag{6}$$

where y_f is the dimensionless coupling constant and f denotes what kind of fermion it is, for instance if $f = e$ we have an electron. Now notice that upon expanding the Higgs field around it's minima we have generated a mass for the fermion:

$$-m_f\bar{\Psi}_f(x)\Psi_f(x)\left(1+\frac{h(x)}{v}\right), \tag{7}$$

where $m_f = y_f/v$.

The presentation here is rather sketchy, but the main point is that explicit mass terms for fermions are not allowed in the standard model due to the gauge symmetry imposed. But by coupling them to the Higgs field which obtain a non–zero vacuum exp. value, we generate such masses while still respecting gauge symmetry!

Problem 4

Calculate the partial decay width for the Higgs boson decaying into a pair of fermions. What is the behaviour for $m_h >> m_f$? How should we modify the result for Higgs decay into a quark, antiquark pair?

Hint 1: Remember that $A^a B_a{}^b C_b = a\ number = C_b A^a B_a{}^b = (CA)_b{}^a B_a{}^b$ =Trace(CAB), where A, C are dim$N$ vectors. CA,B are $N \times N$ matrices and a, b refer to their components ($a, b = 1, \ldots, N$). Upper index refers to a row, and lower index column. Trace$M = M_a{}^a$ i.e. sum over diagonal elements.

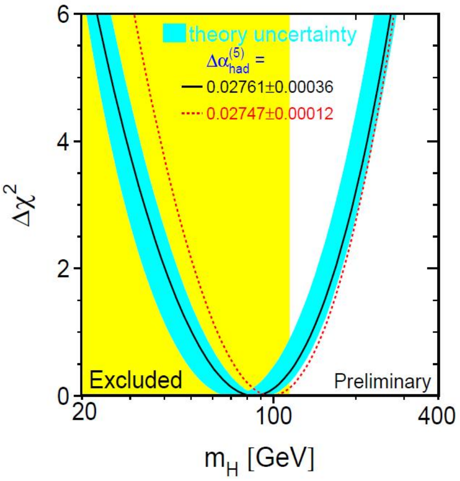
Hint 2: Quarks have one additional quantum number, which the matrix element does not depend on.

Problem 5

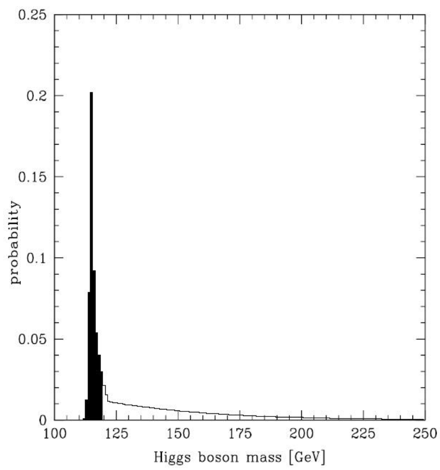
Present the results graphically from problem 3 and 4. One figure should show the **branching ratios** for the different decay modes of the Higgs boson as a function of its mass. The other one should show the **total decay width** of the Higgs boson as a function of its mass. You will need to use computer software for this. Consider m_H from 1 to 500 GeV and exclude the lightest fermion generations if you find the procedure involved and the graphs too messy. Also you will need to use $\log_{10}$ scale. The vacuum energy of the Higgs field is $v = 246$ GeV.

A EWPT of the Higgs Boson mass

We will plot for Higgs boson masses between 1 and 500 GeV, however, searches have constrained the mass to be below 500 GeV, here I give you the probability graph for Higgs boson mass. They show the χ^2 value one get when fitting Standard Model observables from experiments to theory, as function of the Higgs boson mass and the probability density function. So as can be seen, we are actually only interested in the region $\sim$ 120-200 GeV for the Standard Model Higgs boson mass.



theory uncertainty
$\Delta\alpha^{(5)}_{had}$ =
0.02761±0.00036
0.02747±0.00012
Excluded
Preliminary
$\Delta\chi^2$
m_H [GeV]



probability
Higgs boson mass [GeV]