2007-05-24

Suggested projects   Group Theory 07

Below is a list of suggested projects. For each project you find a short description, some suggestions what you could do and some references in cases where you cannot find the material in standard books on group theory. If you have your own suggestion for a project please contact one of us. For each project you also see which one of us that is responsible.

Our intention is that you should work on the projects in groups of two. If this is not convenient for you please contact one of us. We will circulate a list with the projects where you can sign up for one of them. If there are more than two persons interested in a given project we will have to solve this in some way such that there is a more or less even distribution on the different projects.

Write a short report together (3-4 pages) and prepare a joint 15 minutes presentation for the other participants preferably using a computer or slides.

1. **Double groups** (LN)
   The ordinary point groups are sub-groups to the continuous groups of proper or improper rotations, O(3) and SO(3), respectively. There exist a homomorphism between SU(2) and SO(3), with the corresponding “4π rotation” of a spinor function. Double groups are discrete sub-groups of SU(2) and homomorphic to the ordinary point groups.
   Task: Describe the nature of double groups and derive their irreducible representations.

   Participant:
   Participant:

2. **Colour groups** (LN)
   If colour (usually black and white) is associated with the “atoms” of “the molecule”, the corresponding symmetry group will have a colour-changing operation. These colour point and space groups are relevant for magnetically ordered materials, where the colour-changing operations corresponds to inversion of the magnetic moment, or correspondingly to a time-reversal operation.
   Task: Describe the nature of colour point groups and describe how to obtain their irreducible representations. Discuss colour space groups.
   Literature: E.g. “Symmetry Principles and Magnetic Symmetry in Solid State Physics” by S.J. Joshua.

   Participant: *Maciek Borysiuk*
   Participant:

3. **The Jahn-Teller effect** (LN)
   The Jahn-Teller theorem states that a highly symmetric molecules are unstable towards a distortion when its electronic structure is degenerate.
   Task: Derive this theorem and discuss its relevance for a few systems.

   Participant: *Sergiu Arapan*
   Participant: *Diana Iusan*
   Participant: *Kim Duckyoung*

4. **Symmetry in spectroscopy** (LN)
   In spectroscopy we usually encounter transitions due to multi-pole terms in an expansion of the radiation field.
   Task: Discuss the group theoretical arguments for different electronic multi-pole transitions for some specific molecule, e.g. NH_3.

   Participant: *Pelle Linusson*
   Participant: *Anders Gaude*

5. **Coupling of angular momenta and 3j symbols** (LN)
   Often we deal with coupling of angular momenta, two or more.
   Task: Discuss the physics leading to coupling of angular momenta, especially the cases of two and three momenta. Describe the 3j symbols and how they enter in coupled angular momenta.
   Literature: E.g. "Angular Momentum in Quantum Mechanics" by A.R. Edmonds.

   Participant:
   Participant:

6. **Permutation group and the representations of many body states** (LN)
   When we deal with a system of several identical particles, we work with a outer direct group of the point group symmetry and the permutation group.
   Task: Discuss the physics of identical spin-1/2 particles in terms of irreducible representations of the permutation group. This demands some deeper study of the permutation group and its representations.

   Participant: *Francesco Cricchio*
   Participant: *Ricardo Bevilaqua*

7. **Hydrogen atom and SO(4)** (LN)
   For an electron in a Coulomb potential, i.e. the hydrogen atom, the symmetry is higher than the assumed SO(3) rotational symmetry, and larger degeneracies than expected is found.
   Task: Derive the SO(4) symmetry for this problem, and discuss the degeneracies in terms of its irreducible representations.
   Literature: E.g. "Symmetry in Physics, part 2" by J.P. Elliot and P.G. Dawber.

   Participant: *Martin Berglund*
   Participant: *Gustav Carlson*

8. **SU(6)** (JR)
   SU(6) (spin and flavour) can be used to describe wave-functions of baryons.
   Task: Study SU(6) (spin and flavour) closer. What are the relevant representations, why is there no flavour-SU(3) singlet state for baryons, what are the wave-functions for the octet baryons (p, n, Σ, Λ, Ξ), how can the magnetic moments be calculated for these, compare with data and determine the constituent masses for u, d and s quarks.
   Literature: for example Elementary Particle Physics by Nachtman and Particle Data Group, `http://pdg.lbl.gov/`

   Participant: *Manuel Meyer*
   Participant: *Christian Pfeifer*
   Participant: *Niklas Hübel*

9. **Mass relations** (JR)
Using group theory one can get relations between the masses of the baryon octet and decuplet as well as the meson nonets.
Task: Study these mass relations. Derive Gell-Mann–Okubos mass formula and apply it to the different multiplets. Why are the mass-splittings much larger between the isospin multiplets compared to within one such multiplett. Determine the mixing angles of the η, η' and ω, ϕ from the masses in the respective nonet. What are the mixing angles? Compare with data.
Literature: Particle Data Group, `http://pdg.lbl.gov/`

   Participant:
   Participant:

10. **Young Tableaux** (JR)
Young Tableaux can be used to make a Clebsch-Gordan decomposition of tensor products in SU(N).
Task: Study the properties of Young Tableaux and how they can be used to make a Clebsch-Gordan decomposition of tensor products in SU(3) and how this generalizes to SU(N).

    Participant: *Moritz Beckman*
    Participant: *Stefan Löffler*

11. **Exotic hadrons** (JR)
Exotic hadrons are those that do not have the quark structure $q\bar{q}$, qqq or $\bar{q}\bar{q}\bar{q}$.
Task: Study the possible representations of exotic hadrons based on group theory. What irreducible representation are obtained for the quark combinations $qq\bar{q}\bar{q}$ (dimesons) and $qqqq\bar{q}$ (penta-quarks). How are these reduced if one assumes that one only has diquarks which are flavour antisymmetric such as $ud - du$ as in the model by Jaffe and Wilczek. How does the physical mesons σ, κ, a_0 and f_0 fit into this picture? How about the θ^+ baryon? Literature: Particle Data Group, `http://pdg.lbl.gov/`

    Participant: *Carl-Oscar Gullström*
    Participant:

12. **Chiral perturbation theory** (JR)
Chiral perturbation theory based on $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ can be used to make predictions of low-energy hadronic processes and also to relate hadron masses and quark masses. Task: Study these relations

    Participant: *Henrik Petterson*
    Participant: *David Duniec*

13. **Grand Unification** (JR)
In Grand Unification theories the strong, electromagnetic and weak forces are described by one gauge group. The unification scale, where all the gauge couplings unify, is typically of order 10^{16} GeV. The simplest examples are SU(5) and SO(10). The enlarged symmetry means that there are additional guage bosons (forces) which for example lead to proton decay, $p \to e^+\pi^0$, at the unification scale.
Task: What are the fundamental representations and how do the standard model particles fit into them. How can the symmetries be broken to the Standard Model one, SU(3) × SU(2) × U(1)

Participant: *Erik Thomé*
Participant: *Peder Eliasson*

14. **Supersymmetry** (JR)
Supersymmetry is an extension of the Poincare symmetry to the maximal space-time symmetry possible. It predicts that for each fermionic degree of freedom there is a corresponding bosonic one. So for example there are two scalar supersymmetric partners to the electron with all quantum numbers the same except the spin.
Task: Describe the no-go theorem by Coleman and Mandula, the supersymmetry algebra, the supersymmetric harmonic oscillator.
Literature: See for example Introduction to Supersymmetry and Supergravity by Peter West and Dynamical Supersymmetry breaking - why and how by Erich Poppitz, hep-ph/9710274

Participant: *Johan Blåbäck*
Participant: