Home Assignment nr 8 — Group Theory 07

SU(3)

a) SU(2) subgroups of SU(3)
Write down the ladderoperators $I_\pm$, $U_\pm$ och $V_\pm$ for the three different SU(2) subgroups of SU(3) corresponding to isospin, U-spin and V-spin. How do they act on a state with weight (μ_1, μ_2).

How are the quark states $|u\rangle$, $|d\rangle$ and $|s\rangle$ modified by these ladderoperators?

Show that I, U and V-spin are SU(2) algebras. In other words find the operators I_3, U_3 and V_3

b) The quadratic Casimir operator of SU(3)
Show that the quadratic Casimiroperator, $C_1 = T_a T_a$ can be written in terms of the Cartan generators and the ladder operators as $T_a T_a = H_1^2 + 2H_1 + H_2^2 + 2I_- I_+ + 2V_- V_+ + 2U_- U_+$. (Remember that $[E_{\vec{\alpha}}, E_{-\vec{\alpha}}] = \vec{\alpha} \cdot \vec{H}$ and $T_1^2 + T_2^2 = I_- I_+ + I_+ I_- = 2I_- I_+ + [I_+, I_-]$ etc.) This expression can be used to calculate the eigenvalues of the quadratic Casimir operator. Apply this operator to the highest weight state of an irreducible representation (m, n) and **show that** $C_1 = (m^2 + mn + n^2)/3 + m + n$
Calculate the quadratic Casimir operators of the following representations, $\mathbf{3}$, $\mathbf{3}^*$, $\mathbf{6}$, $\mathbf{8}$ och $\mathbf{10}$.

c) Clebsch-Gordan decomposition of tensor products
Perform the multiplications $\mathbf{3} \otimes \mathbf{3}^* = \mathbf{1} \oplus \mathbf{8}$ och $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$ using the graphical method.