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Chiral perturbation theory

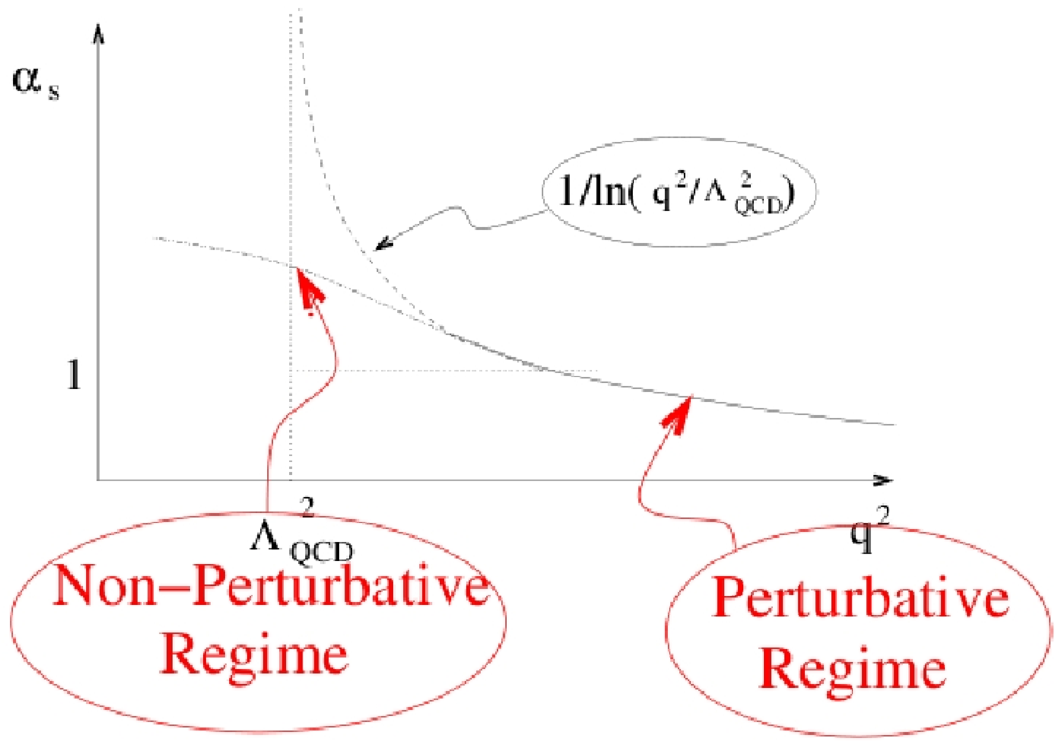
or

Who needs quarks anyway?

David Duniec & Henrik Pettersson

Non-perturbative QCD

The theory for the strong interaction, QCD, can not be used in the non-perturbative region, where mesons and baryons are the degrees of freedom.



C. Hanhart, CSS06 lectures

The "running" of α_s.

Non-perturbative QCD II

What can we use instead to obtain predictions and test our understanding of the strong interaction?

Lattice QCD, quark models and *effective theories*.

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Effective theories I

What is meant by an "effective theory"? – use only relevant degrees of freedom and ignore the rest (usually "short-range interactions" and the like). Examples:

- Fermi theory of β decay
- BCS theory of superconductivity
- heavy quark EFT
- non-relativistic QCD

etc.

Effective theories II

In particle physics, effective theories are low-energy approximations of fundamental theories.

Typically, the range of validity is given by some momentum scale Λ and one may expand interactions in p/Λ or some other small parameter. Terms in this expansion have parameters which can be calculated from the underlying theory (not in our case of course, if we could do that we would just do it) or determined from experiment.

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How to make and use an EFT

1. Write down the most general Lagrangian that respects the symmetries and other properties of the underlying theory.

2. Respect the range of validity of the effective theory. You may then study the underlying theory indirectly.

Chiral Perturbation Theory

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The EFT in this case is Chiral Perturbation Theory (often abbreviated χPT).

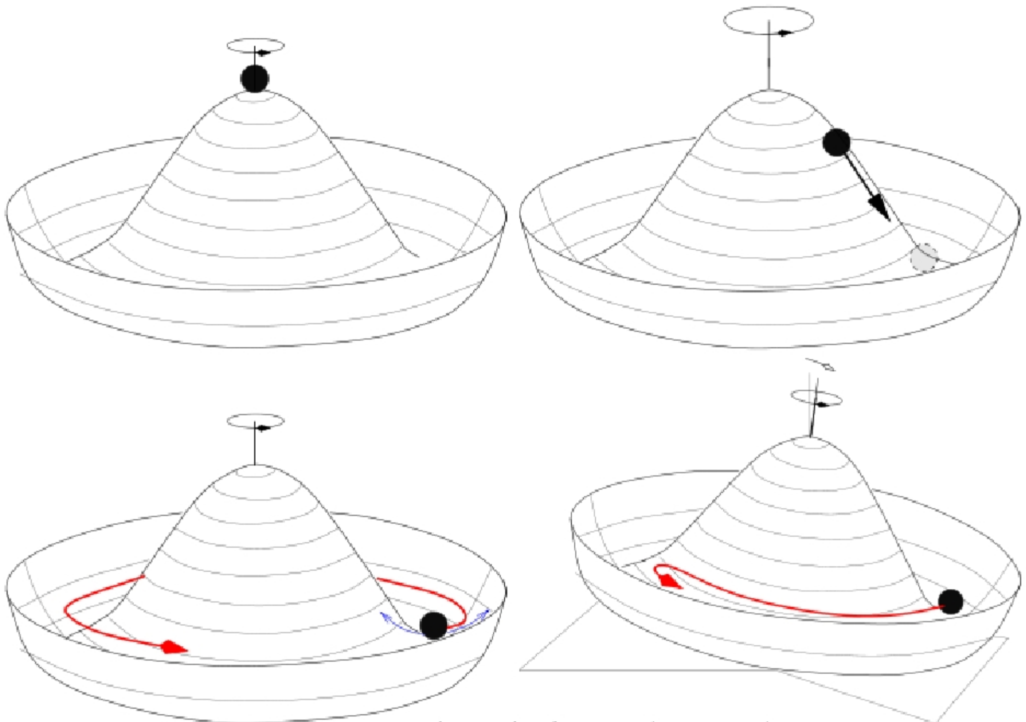
The light-flavour Lagrangian

$$\mathcal{L}^0_{QCD} = \sum_{l=u,d,s} \{\bar{q}_{R,l} i \not{D} q_{R,l} + \bar{q}_{L,l} i \not{D} q_{L,l}\} - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}^{\mu\nu}_a$$

would have a SU(3)$_A \times$SU(3)$_V \times$U(1)$\times$U(2) symmetry if quark masses vanished.

Spontaneous symmetry breaking I

The chiral symmetry is spontaneously broken (=the axial charge doesn't annihilate the vacuum) ⇒ Goldstone bosons appear. Since there are eight charges, one expects eight GB.



Picture taken from C.Hanhart, CSS06 lectures

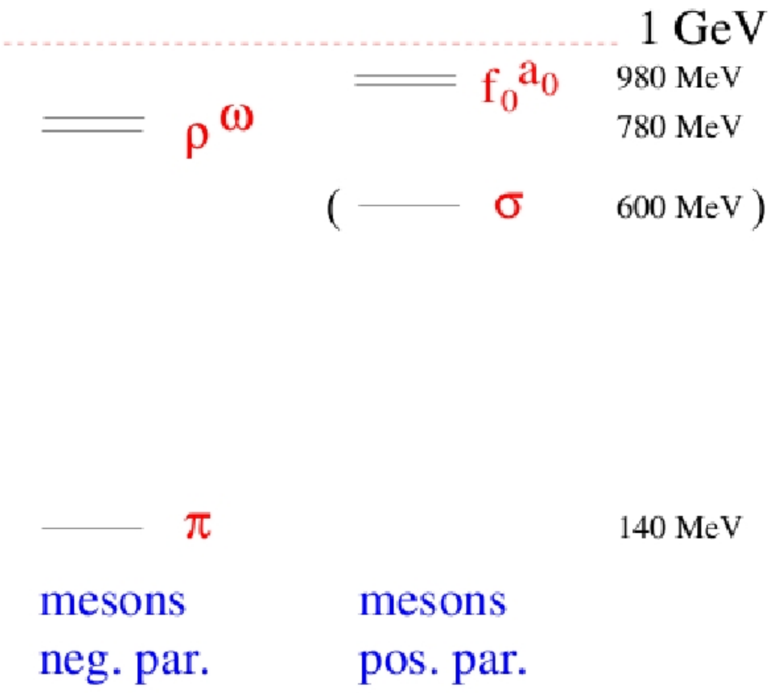
The usual "Mexican hat". The lower right potential illustrates the situation corresponding to physical pseudoscalar mesons.

SSB II

Why does one think there is SSB in QCD? The QCD Lagrangian has $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times \mathrm{U}(1)_V$ symmetry in the chiral limit. Then one expects multiplets according to the irreducible representations of that group. Parity doublets are expected, but not observed.

Pions are very light

Second hint to symmetry breaking: the pseudoscalar mesons are light compared to the vector mesons.



Some meson masses.

Also, the pions are extra light (isospin less broken than SU(3)).

Representing the symmetry

A non-linear representation of the SU(3) group is given by

$$U(\Theta) = \exp\left(-i\sum_{a=1}^{8}\Theta_a\frac{\lambda_a}{2}\right)$$

(λ_a are the hermitian Gell-Mann matrices).

For the mesons, we have an $3 \otimes \bar{3} = 8 \oplus 1$ representation. $U(x)$ can be written in terms of its dynamical variables in this representation:

$$U(x) = \exp\left(i\frac{\phi(x)}{F_0}\right), \quad \phi(x) = \textstyle\sum_{a=1}^{8}\lambda_a\phi_a(x) \equiv$$

$$\begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{(2)}\pi^+ & \sqrt{(2)}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

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Finding the effective Lagrangian

Remember, the goal was to write the most general Lagrangian that respects all symmetries.

The most general term one can write for massless quarks is

$$\mathcal{L}_{\text{eff}} = \frac{F_0^2}{4}\text{Tr}(\partial_\mu U \partial^\mu U^\dagger).$$

Expanding $U = 1 + i\phi/F_0 + \cdots$, $\partial_\mu U = i\partial_\mu\phi/F_0 + \cdots$ and putting that in $\mathcal{L}_{\text{eff}}$ gives

$$\mathcal{L}_{\text{eff}} = \frac{F_0^2}{4}\text{Tr}\left[\frac{i\partial_\mu\phi}{F_0}\left(-\frac{i\partial^\mu\phi}{F_0}\right)\right] + \cdots = \frac{1}{4}\text{Tr}(\lambda_a\partial_\mu\phi_a\lambda_b\partial^\mu\phi_b) + \cdots =$$
$$\frac{1}{4}\partial_\mu\phi_a\partial^\mu\phi_b\text{Tr}(\lambda_a\lambda_b) + \cdots = \frac{1}{2}\partial_\mu\phi_a\partial^\mu\phi_a + \mathcal{L}_{\text{int}}$$

From the absence of a two-ϕ term one sees that the Goldstone fields are massless, as expected.

The effective Lagrangian II

However, we know that there is explicit breaking due to the quark masses. Introducing

$$\mathcal{L}_M = -\bar{q}_R M q_L - \bar{q}_L M^\dagger q_R, \; M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}.$$

In fact, $\mathcal{L}_M$ would be invariant if $M \mapsto RML^\dagger$. To lowest order in M, most general Lagrangian $\mathcal{L}(U, M)$ invariant under $U(x) \mapsto RU(x)L^\dagger$ and $M \mapsto RML^\dagger$ becomes: $\mathcal{L}_{\text{s.b.}} = \frac{F_0^2 B_0}{2} \text{Tr}(MU^\dagger + UM^\dagger)$.

The terms of second order in $\mathcal{L}_{\text{s.b.}}$ are $\mathcal{L}_{\text{s.b.}} = -\frac{B_0}{2}\text{Tr}(\phi^2 M) + \cdots$

$$\begin{aligned}\text{Tr}(\phi^2 M) = 2(m_u + m_d)\pi^+\pi^- + 2(m_u + m_s)K^+K^- + \\ 2(m_d + m_s)K^0\bar{K}^0 + (m_u + m_d)\pi^0\pi^0 \\ + \frac{2}{\sqrt{3}}(m_u - m_d)\pi^0\eta + \frac{m_u + m_d + 4m_s}{3}\eta^2\end{aligned}$$

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The Gell-Mann–Okubo relation

If one neglects the light quark mass difference (so that there isn't any $\pi^0 - \eta$-mixing) one gets

$$\begin{aligned} M_\pi^2 &= 2B_0 m \\ M_K^2 &= B_0(m + m_s) \\ M_\eta^2 &= \frac{2}{3}B_0(m + 2m_s) \end{aligned}$$

and these masses satisfy the Gell-Mann–Okubo relation

$$4M_K^2 = 4B_0(m + m_s) = 2B_0(m + 2m_s) + 2B_0 m = 3M_\eta^2 + M_\pi^2$$

independently of B_0.

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Quark mass ratios

From the relations above, one cannot, without knowledge of B_0, extract quark masses explicitly. However, one can extract quark mass ratios:

$$\frac{M_K^2}{M_\pi^2} = \frac{m+m_s}{2m} \Rightarrow \frac{m_s}{m} = 25.9$$
$$\frac{M_\eta^2}{M_\pi^2} = \frac{2m_s+m}{3m} \Rightarrow \frac{m_s}{m} = 24.3$$

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References:

- Jones, H. F.: *Groups, Representations and Physics*, Taylor & Francis, NY 1998.
- Hanhart, C.: *From QCD to Hadron Physics*, presentation slides from CSS2006.
- Scherer, S. and Schindler, M.: *A Perturbation Theory Primer*, arXiv:hep-ph/0505265v1

Extra slide I – Another mass ratio

$$\frac{m_u}{m_d} = \frac{2M^2_{\pi^0} - M^2_{K^0} + M^2_{K^+} - M^2_{\pi^+}}{M^2_{K^0} - M^2_{K^+} + M^2_{\pi^+}} = 0.55$$

Extra slide II – Implications for scattering

The Goldstone theorem also states that the GB don't interact at low ("vanishing") momenta. One would like to test that in e.g. $\pi - \eta$ scattering, but it's impossible to create η-beams. One can use crossing and use η-decays like $\eta \longrightarrow \pi^+\pi^-\pi^0$ and $\eta \longrightarrow \pi^0\pi^0\pi^0$.

The amplitude for the first decay can be written (Sutherland, Gasser, Leutwyler)

$$A(s,t,u) = \frac{B_0(m_u - m_d)}{3\sqrt{3}f_\pi^2}\left\{1 + \frac{2s-t-u}{m_\eta^2 - m_\pi^2}\right\} = \frac{1}{Q^2}\frac{m_K^2}{m_\pi^2}(m_\pi^2 - m_K^2)\frac{M(s,t,u)}{3\sqrt{3}f_\pi^2};$$

here, $s = (p_\eta - p_\pi^0)^2; t = (p_\eta - p_\pi^+)^2; u = (p_\eta - p_\pi^-)^2$,
$M(s,t,u) = \frac{3s-4m_\pi^2}{m_\eta^2 - m_\pi^2}$ and
$Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} = \frac{m_K^2}{m_\pi^2}\frac{m_K^2 - m_\pi^2}{(m_{K^0}^2 - m_{K^+}^2)^{\text{strong}}}(1 + \mathcal{O}(m_{\text{quark}}^2))$.

Thus the decay width for the η-decay will give a measurement of the light quark mass difference (or, more correctly $\frac{m_u - m_s}{m_s}$).