Supersymmetry

- Coleman-Mandula theorem
- The sypersymmetry algebra
- The supersymmetric harmonic oscillator

What we want to accomplish

- Construct a supersymmetry group
- Study its algebra
- See if there are any physical consequences

The Coleman-Mandula theorem

1. The supersymmetry group has a subgroup locally isomorphic to the Poincaré group
2. There is a finite number of particles beneath any given upper limit mass
3. Continous energy spectra in scattering
4. Continous scattering angles

Then the supersymmetry group is locally isomorphic to the direct product of the Poincaré group P and a Lorentz invariant compact group T

Gradation and odd generators

Even	Odd
$P_a, \; L_{ab}, \; T_s$	Q^i_α

$$
\begin{aligned}
[\text{even}, \text{even}] &= \text{even} \\
\{\text{odd}, \text{odd}\} &= \text{even} \\
[\text{even}, \text{odd}] &= \text{odd}
\end{aligned}
$$

$$
\begin{aligned}
\left[Q^i_\alpha, L_{ab}\right] &= (a_{ab})^\beta_\alpha Q^i_\beta \\
\left[Q^i_\alpha, P_a\right] &= (b_a)^\beta_\alpha Q^i_\beta \\
\left[Q^i_\alpha, T_s\right] &= (c_s)^{\beta i}_{\alpha j} Q^j_\beta \\
\left\{Q^i_\alpha, Q^j_\beta\right\} &= \text{Combination of even generators}
\end{aligned}
$$

The supersymmetry algebra

$$
\begin{aligned}
[Q_\alpha, L_{ab}] &= \frac{1}{2}(\sigma_{ab})_\alpha^{\ \beta} Q_\beta \\
\left[Q_\alpha^i, P_a\right] &= 0 \\
[Q_\alpha, T_s] &= i(\gamma_5)_\alpha^{\ \beta} Q_\beta \\
\{Q_\alpha, Q_\beta\} &= 2(\gamma^a C)_{\alpha\beta} P_a
\end{aligned}
$$

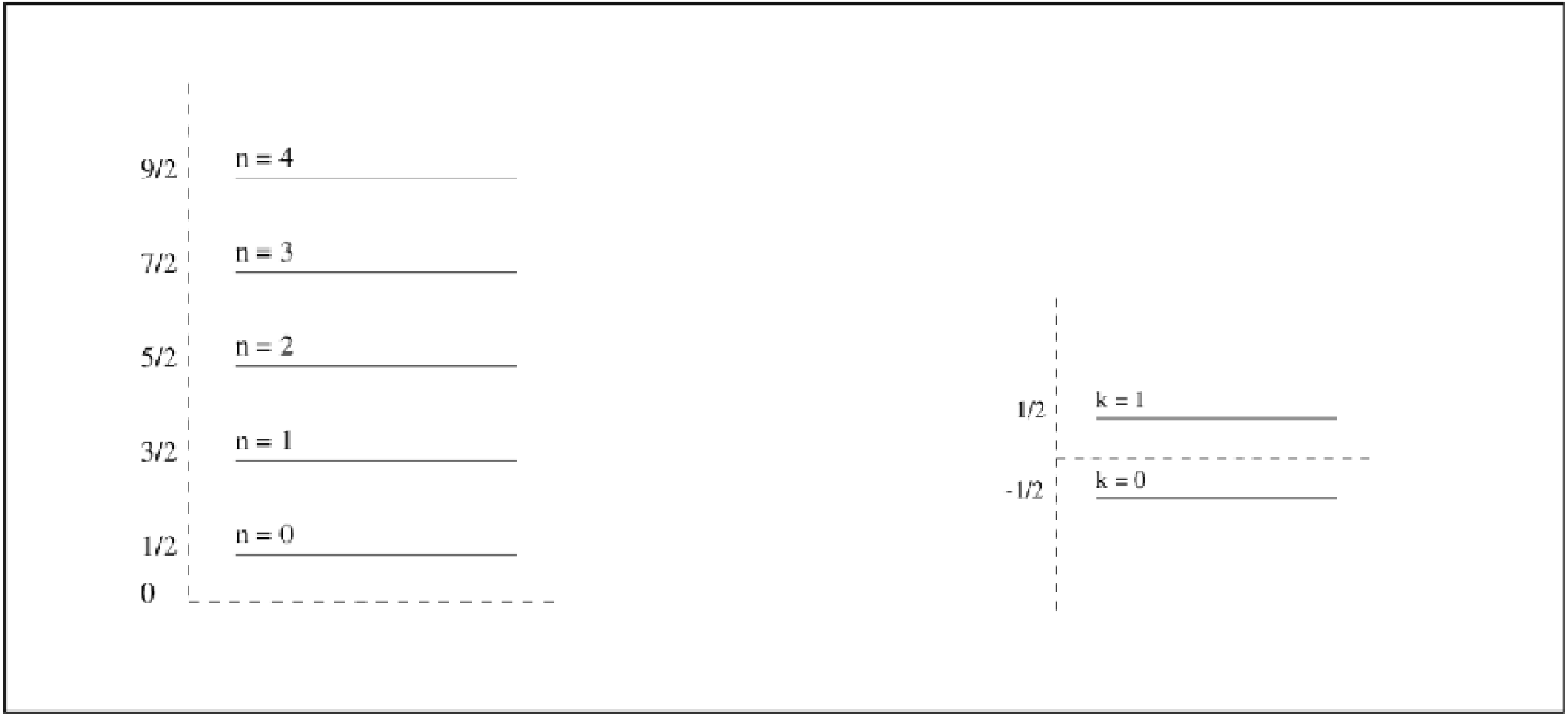
The quantum mechanical oscillators

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2 x^2}{2}$$

$$\hat{H}_B = \hbar\omega\left(b^\dagger b + \frac{1}{2}\right)$$

$$\hat{H}_F = \hbar\omega\left(f^\dagger f - \frac{1}{2}\right)$$

The quantum mechanical oscillators



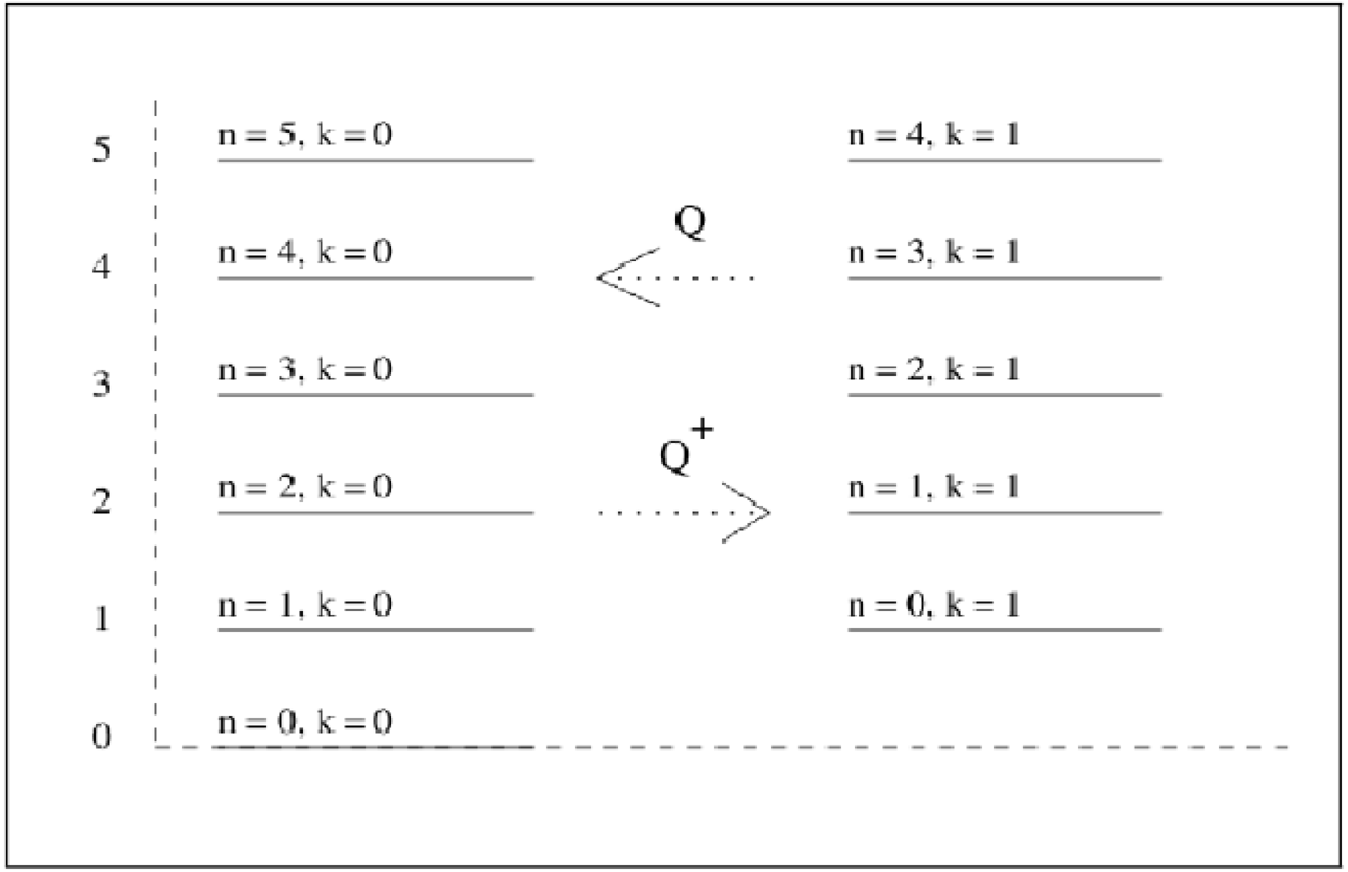
$$\hbar\omega\left(b^\dagger b + \frac{1}{2}\right) \mid n \rangle = \hbar\omega\left(n + \frac{1}{2}\right) \mid n \rangle\,, \qquad \hbar\omega\left(f^\dagger f - \frac{1}{2}\right) \mid k \rangle = \hbar\omega\left(k - \frac{1}{2}\right) \mid k \rangle$$

The supersymmetric oscillator

$$Q = b^\dagger f, \quad Q^\dagger = f^\dagger b.$$

$$\hat{H} = \hat{H}_B + \hat{H}_F = \hbar\omega\left(b^\dagger b + f^\dagger f\right) = \hbar\omega\left\{Q^\dagger, Q\right\}$$

The supersymmetric oscillator



$$\hat{H} | \, k, n \, \rangle = \hbar\omega(n+k) | \, k, n \, \rangle \, .$$