Young Tableaus

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June 5, 2007

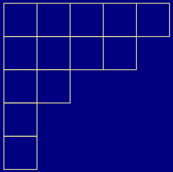
Introduction

- For infinite groups it is easier to work with tensors than directly with representations.
- Decomposition into symmetric and antisymmetric parts
- Graphical representation by Young tableaus

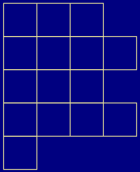
General Rules

Tensor with n indices $\longrightarrow$ arrangement with n boxes

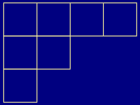
1. Rows are left aligned
2. Each row may not contain more boxes than the row above
3. For SU(N): each column may not contain more than N boxes



, but not



Dimensions



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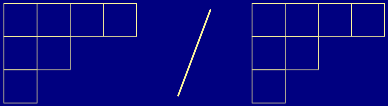

- Numerator: Put N in the upper left box. The numbers for the adjacent boxes are decreased by one for moving down and increased by one for moving right
- Denominator: Fill each box with the number of boxes below and to the right (only same column/row) plus one
- Multiply all numbers in the nominator and divide them by the product of all numbers in the denominator.

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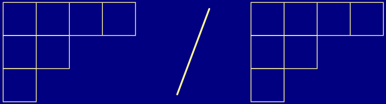

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$$N = 3: \begin{array}{|c|c|c|c|}\hline 3 & 4 & 5 & 6 \\ \hline 2 & 3 \\ \cline{1-2} 1 \\ \cline{1-1}\end{array} \Bigg/ \begin{array}{|c|c|c|c|}\hline 6 & 4 & 2 & 1 \\ \hline 3 & 1 \\ \cline{1-2} 1 \\ \cline{1-1}\end{array} = \frac{2\cdot 3\cdot 3\cdot 4\cdot 5\cdot 6}{2\cdot 3\cdot 4\cdot 6} = 15$$

Decomposition of tensor products

$$x \otimes y = ? \oplus ? \oplus \ldots \oplus ?$$

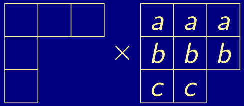
Example: SU(3)

$$8 \otimes 8 = \yng(2,1) \otimes \yng(2,1) = ?$$

Decomposition of tensor products

Rules for decomposition

1. Fill each box of the right tableau T_R with a label (e.g. $a, b, c, \ldots$) identifying the row



2. Add the a's to T_L, such that
   - T_L is a legal Young tableau at any time
   - there is only one a in each column (antisymmetry)
   - the sequence of labels reading from right to left and then top to bottom is *admissible*, i.e. $\sharp(a) \geq \sharp(b) \geq \ldots$, at any position and any time

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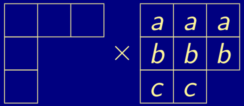

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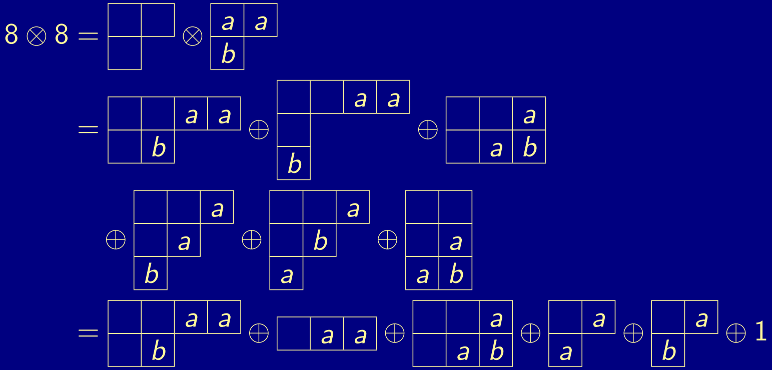
3. Two tableaus of same shape are counted twice only if the labels are distributed differently
4. Remove all columns with N boxes (trivial repr. of SU(N))
5. Continue with the b's etc. according to the same rules

Decomposition of tensor products



$$= 27 \oplus 10 \oplus \overline{10} \oplus 8 \oplus 8 \oplus 1$$

Summary

- Young tableaus represent the relevant properties of tensors very concise and
- allow to calculate the dimensions of the corresponding representations and
- the decomposition into fundamental representations according to simple rules.

Many thanks for your attention!

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References:

- H.F. Jones, *Groups, Representations and Physics*, 2^{nd} edition, Taylor & Francis.
- *SU(n) Mulitplets and Young Diagrams* from D.E. Groom *et al.*, *The European Physical Journal* **C15** *(2000) 1 and 2001 off-year partial update for the 2002 edition available on the PDG WWW pages (URL: http://pdg.lbl.gov/)*