Quantum chromodynamics

and colour singlet exchange in high energy interactions

Rikard Enberg

ISV seminar, May 9, 2003

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Overview

- Quantum Chromodynamics (QCD)
  - What is QCD?
  - Why QCD?
- Colour singlet exchange
  - Gluon ladders and BFKL
  - High energy scattering
- Gaps between jets
  - Monte Carlo simulation
- Vector mesons
  - Meson wave functions

Particle physics: components

Charge	Generation		
	I	II	III
$+\frac{2}{3}$	u up	c charm	t top
$-\frac{1}{3}$	d down	s strange	b bottom
-1	e^-	μ^-	τ^-
0	ν_e	ν_μ	ν_τ

$\leftarrow$ quarks

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← quarks (charge $+\frac{2}{3}$ and $-\frac{1}{3}$ rows)

← leptons (charge -1 and 0 rows)

Gauge bosons

	Electromagnetic interaction	Strong interaction	Weak interaction		
Boson	γ photon	g gluon	Z^0	W^+	W^-
El. charge	0	0	0	$+1$	-1
Other charge	none	colour	weak isospin		
Couples to	el. charge	colour	weak isospin		

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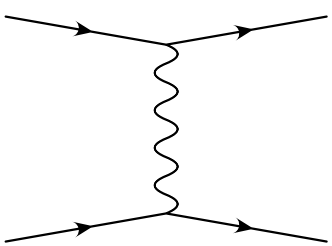

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QCD is a Quantum Field Theory similar to QED:

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fermions:	electrons	$\longrightarrow$	quarks
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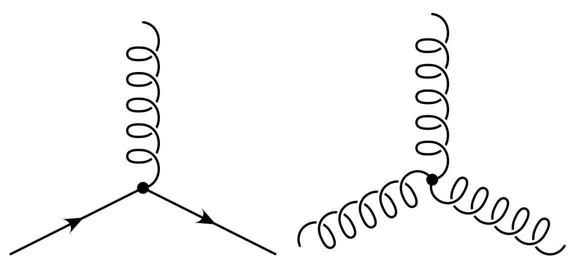

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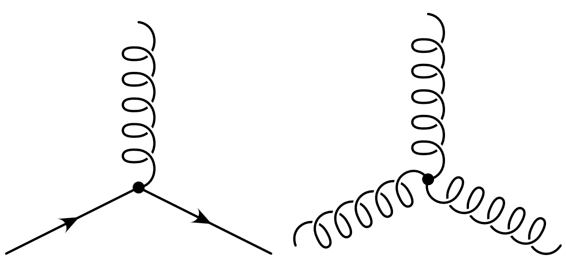

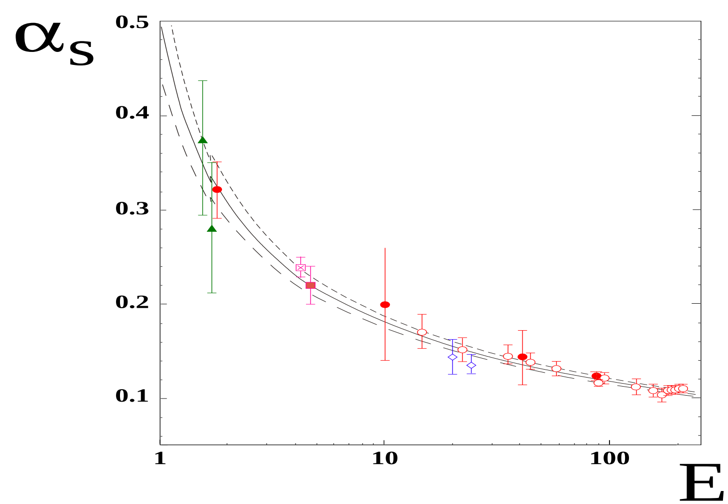

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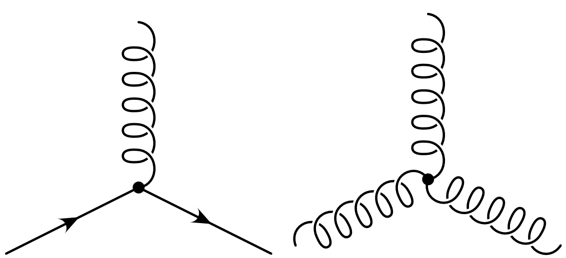

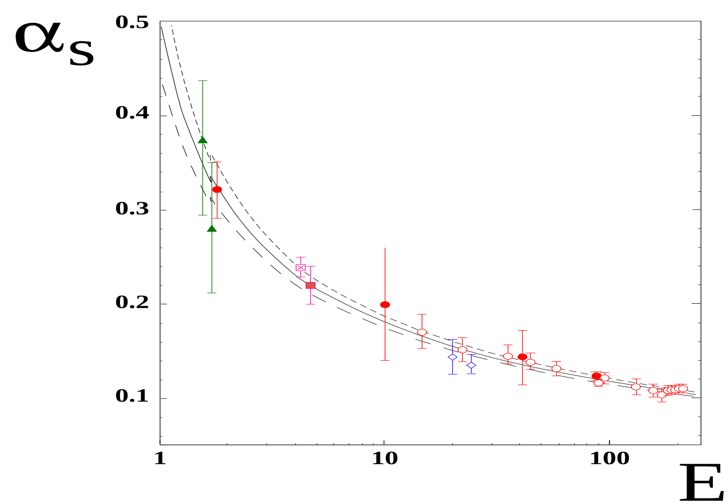

(asymptotic freedom & confinement)

QCD, short intro

The QCD Lagrangian

(q_f are the quark fields, A^a_μ gluon fields)

$$\mathscr{L}_{\rm QCD} = \sum_{f=1}^{6} \bar{q}_f (i\gamma^\mu D_\mu - m_f) q_f - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

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$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

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&+ \quad g f^{abc} A^b_\mu A^c_\nu \, \partial^\mu A^{a\nu} - \frac{g^2}{4} f^{abc} f^{ade} A^b_\mu A^c_\nu A^{d\mu} A^{e\nu}
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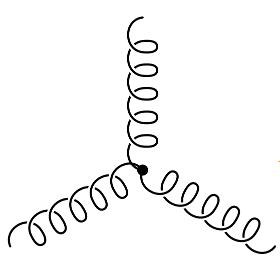

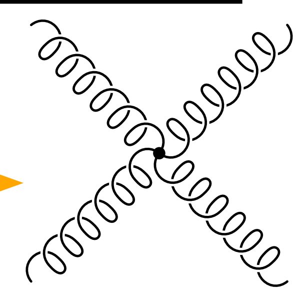

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  - ☞ proton, QCD corrections, hadronisation. . .

Rapidity gaps?

In hadronic collisions, we define the quantities rapidity (y) or pseudorapidity (η):

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} \approx - \ln \tan \frac{\theta}{2} = \eta$$

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Rapidity gaps arise from colour singlet exchange!

Colour singlet?

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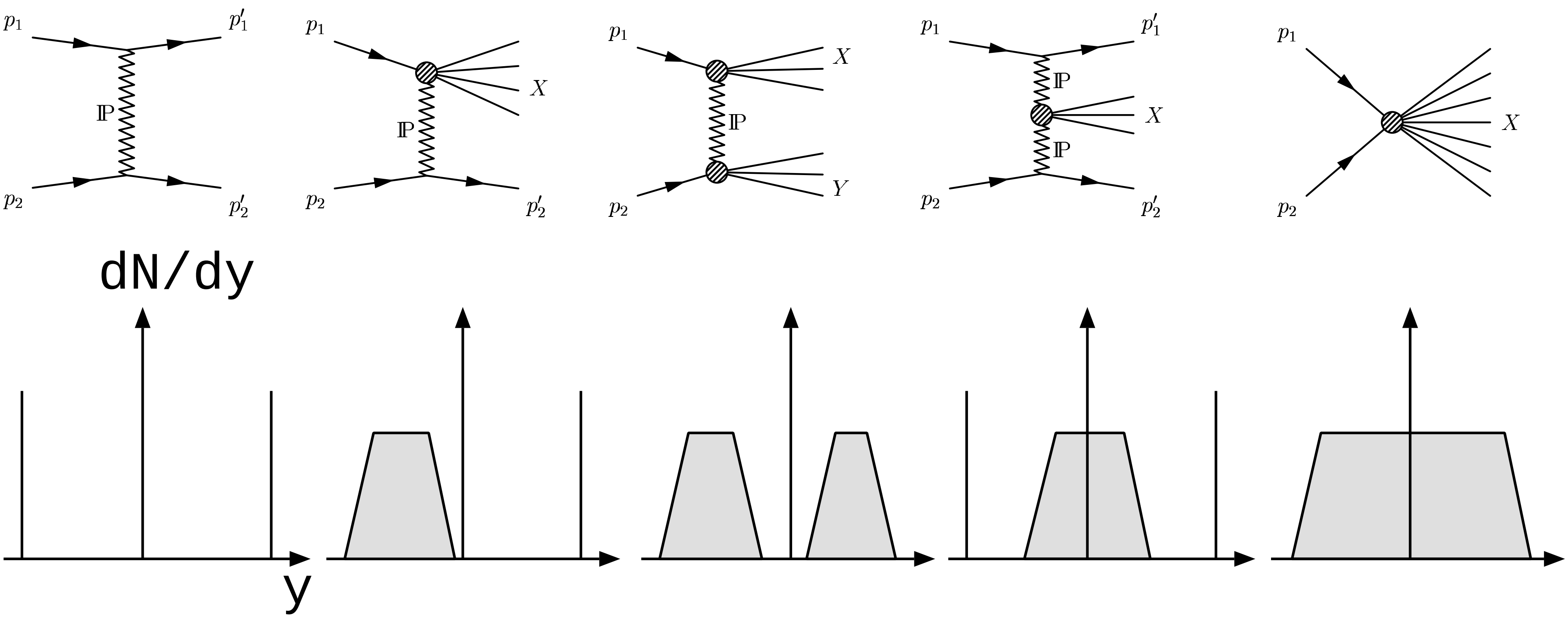
- so exchange of several gluons can lead to colour singlet exchange

  $\Rightarrow$ no colour connection between the quarks

  $\Rightarrow$ no colour field, no hadronisation products

Rapidity gaps experimentally



elastic scattering

single diffraction

double diffraction or gaps between jets

DPE (double pomeron)

inelastic scattering

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- *Hard colour singlet exchange:* events have a central rapidity gap between two high-p_T systems ($|t|$ is large)
- These kinds of gaps are probably of different origin

The thesis

In the thesis, I have studied

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  ⇒ Perturbative “hard” colour singlet exchange. . .

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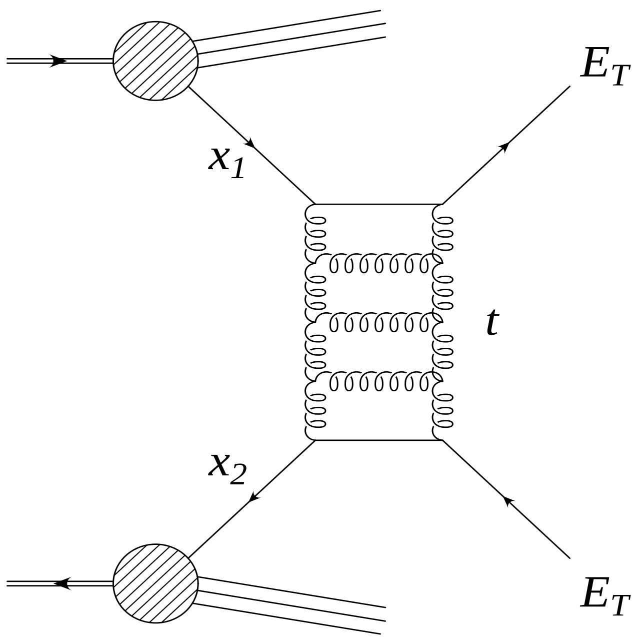
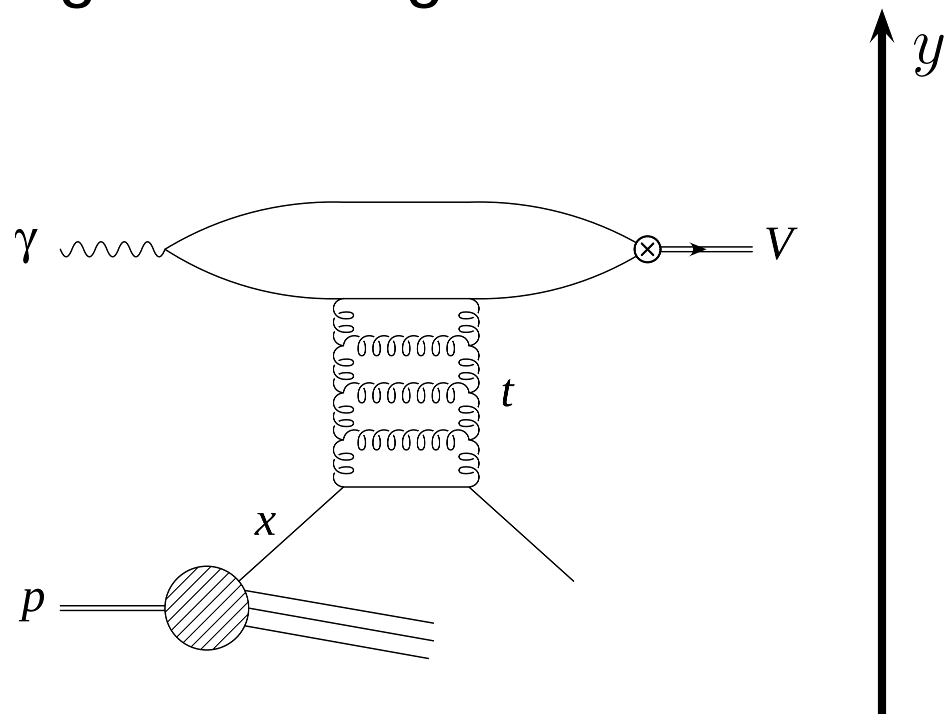
Hard colour singlet exchange

This talk is about hard colour singlet exchange. . .





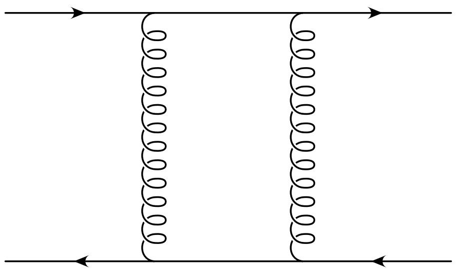
Gaps between jets
in $p\bar{p}$ @ 2 TeV

Vector meson production
in γp @ 100 GeV

Same basic mechanism!

Hard colour singlet exchange

Consider the scattering of two quarks in perturbative QCD where no colour is exchanged in the t-channel. The lowest order process is



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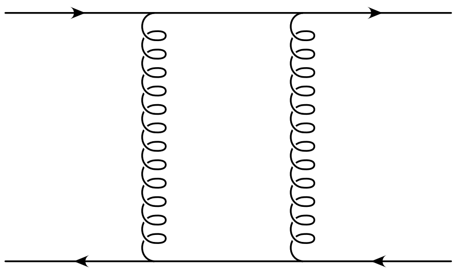

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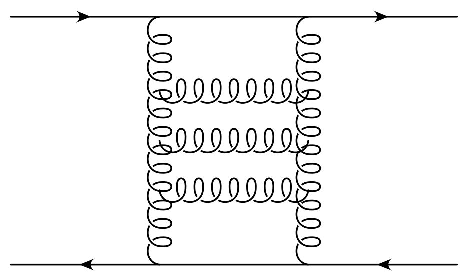
…and we can add rungs to form a gluon ladder:



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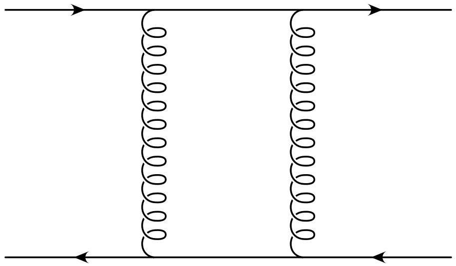

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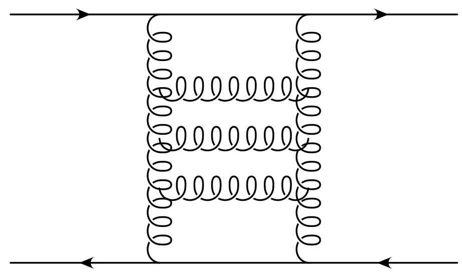
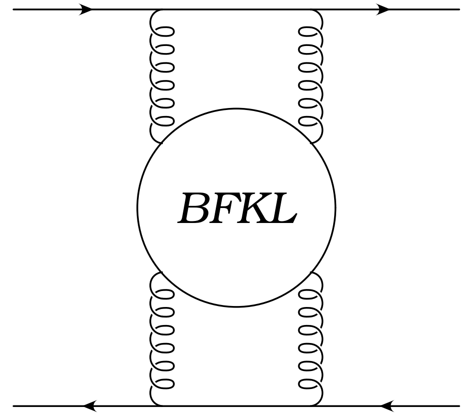


Important effect in the high energy (Regge) limit, $s/|t| \gg 1$.

$\rightarrow$ Amplitude dominated by terms $\propto [\alpha_s \ln(s/|t|)]^n$

The BFKL equation resums these terms to all orders:

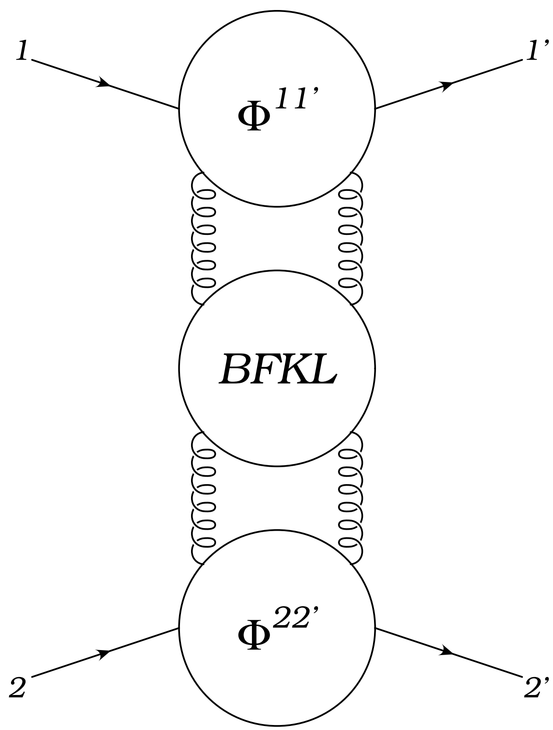


Includes virtual corrections $\rightarrow$ reggeization of gluons.

The BFKL equation is an evolution equation in $y \sim \ln(\hat{s}/|t|)$ (or in $1/x$).
Cf. DGLAP, which is an evolution eqn. in Q^2.

Setup – BFKL Calculation

For the scattering amplitude, we need 3 pieces:



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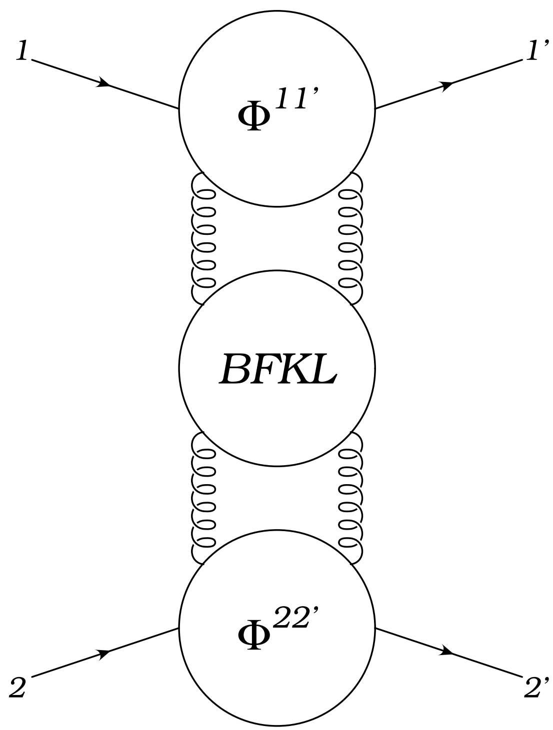

K_{BFKL}: evolution of ladder

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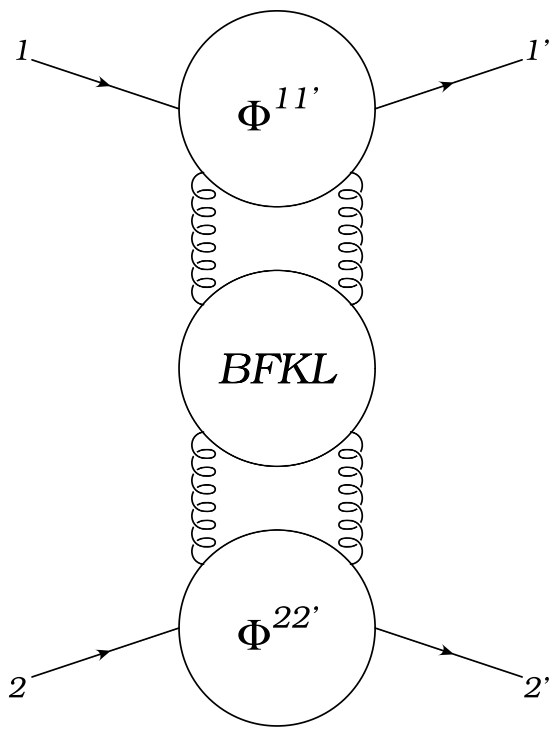

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K_{BFKL}: evolution of ladder
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(Balitsky, Fadin, Kuraev & Lipatov)

$\Phi^{1\to1'}$: impact factor for transition
$1 \to 1'$ coupled to pomeron
from Feynman diagrams
(and VM wave function. . .)

Impact factors & conformal spin

Lipatov solved the BFKL eqn for $|t| > 0$ using conformal symmetry of the kernel

$\rightarrow$ expansion in a basis of conformal eigenfunctions $E_{n,\nu}$

$$\mathcal{A} \propto \sum_n \int d\nu \, \frac{\left(\nu^2 + \frac{n^2}{4}\right) e^{\omega_n(\nu) y}}{\left[\nu^2 + (\frac{n-1}{2})^2\right] \left[\nu^2 + (\frac{n+1}{2})^2\right]} \, {I^1}_{n,\nu}(\boldsymbol{k}, \boldsymbol{q}) \, {I^2_{n,\nu}}^{\star}(\boldsymbol{k}', \boldsymbol{q})$$

n	conformal spin
$\omega_n(\nu)$	eigenvalues of the BFKL kernel
$I^{1,2}_{n,\nu}$	projections of the impact factors $\Phi^{1,2}(\boldsymbol{k}, \boldsymbol{q})$ on the BFKL conformal eigenfunctions $E_{n,v}$

Eh, conformal invariance...?

Conformal invariance of the BFKL equation:

- High energy limit: integrals over $d^4k = dk_+ dk_- d^2\boldsymbol{k}$, integrate out k_+ and k_-
  $\Rightarrow$ all dynamics in transverse momentum plane, with 2-dim vectors $\boldsymbol{k}$

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  $\Rightarrow$ represent as complex numbers $\rho = \rho_x + i\rho_y$
- Then the equation is invariant under transformations

$$\rho \mapsto \frac{a\rho + b}{c\rho + d} \quad \text{where} \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2, \mathbb{C})$$

Mueller–Tang approximation

Conformal spin n: integer labelling eigenfunctions

- In the sum over n in the expansion, the $n = 0$ term dominates strongly for large rapidity y ($\mathcal{A}_{n\neq 0} \to 0$ as $y \to \infty$)

  $\Rightarrow$ amplitude usually approximated by $n = 0$ component

- This is known as the Mueller–Tang approximation

- In quark–quark scattering the formula simplifies a lot

  $\rightarrow$ basis for gaps between jets

Gaps between jets @ Tevatron

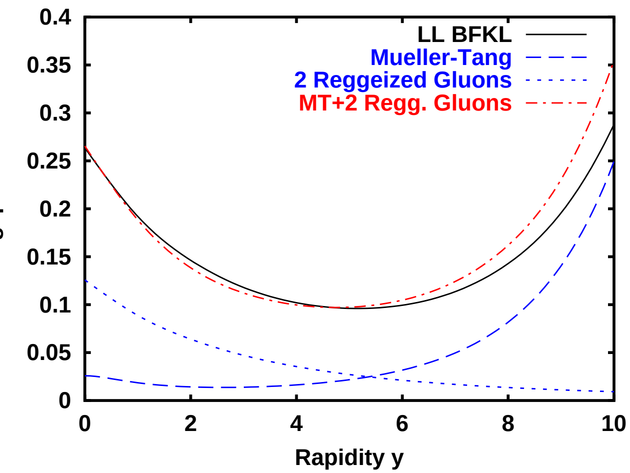
[PLB 524 (2002) 273, with G. Ingelman & L. Motyka]

- Mueller–Tang approx. doesn't agree with DØ data
- Next-to-leading logarithmic (NLL) BFKL effects are known to be important and are not included in Mueller–Tang or Lipatov's solution

> Therefore, we have solved the BFKL equation numerically, (not using Lipatov's solution)

Parton–parton results

Gap fractions, (quark–quark, BFKL / LO QCD):



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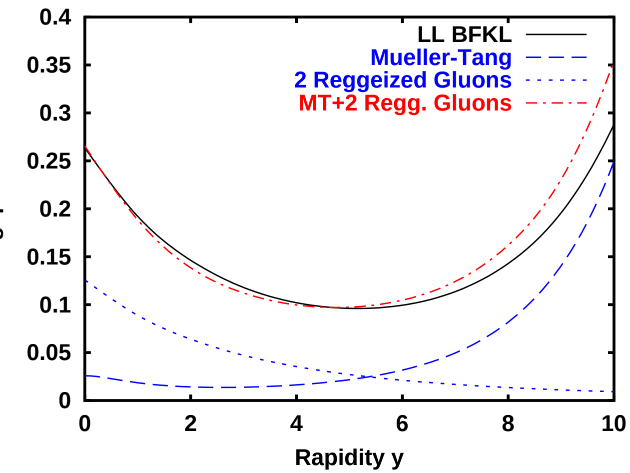

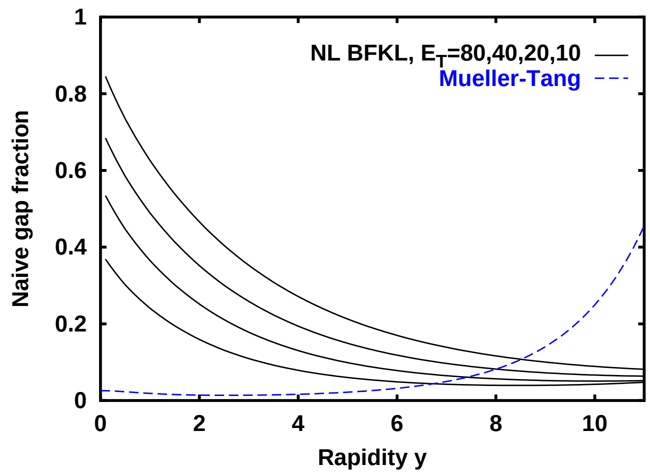

Parton–parton results

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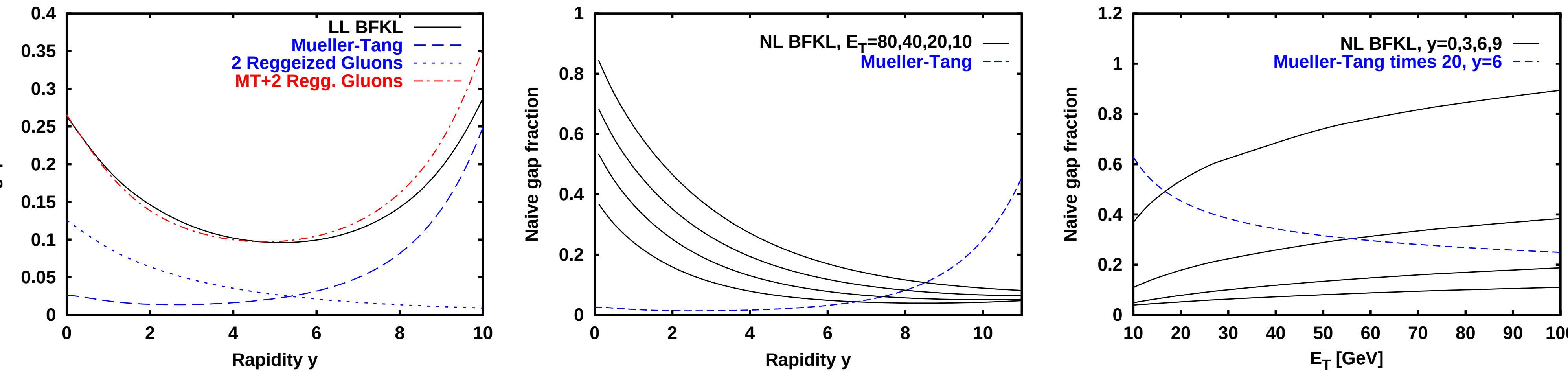

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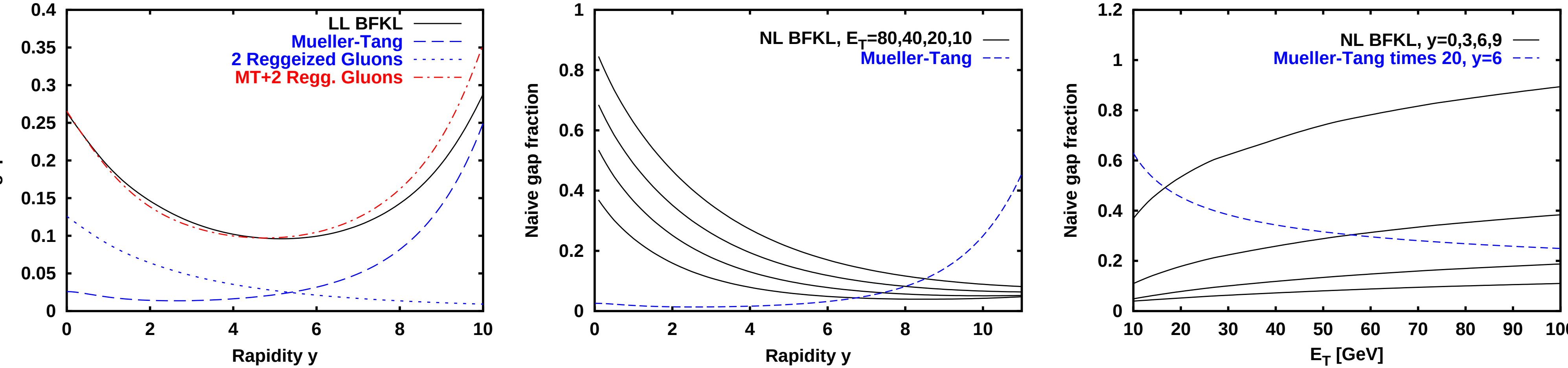

Different behaviour from MT — Note the NLL influence !

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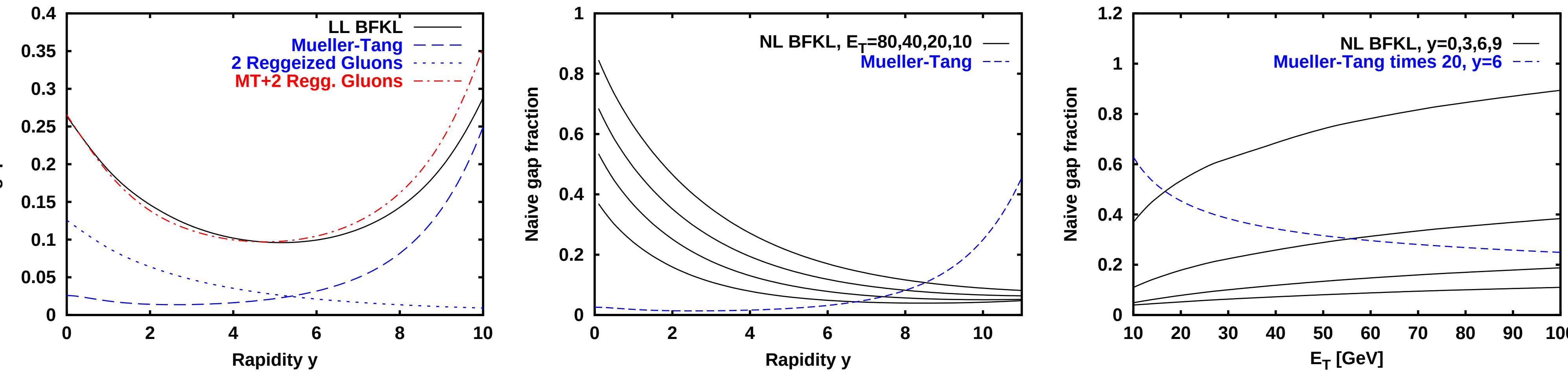


Different behaviour from MT — Note the NLL influence !

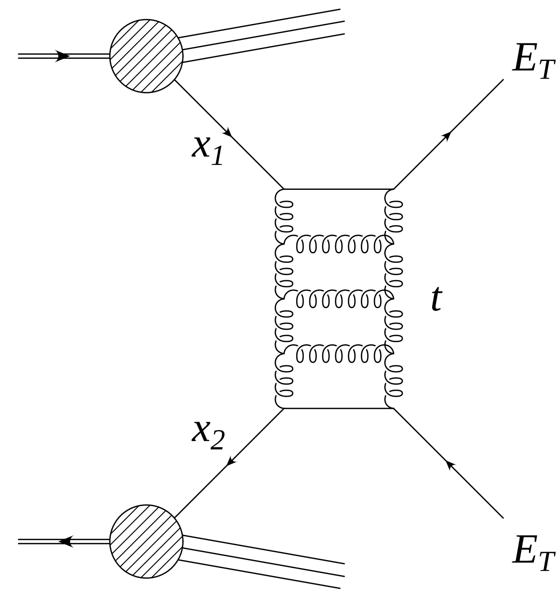
Why this difference?

- $n \neq 0$ contribution dominates for small–moderate y
- BFKL evolution does not reach asymptotic region in y

Comparison to data — PYTHIA

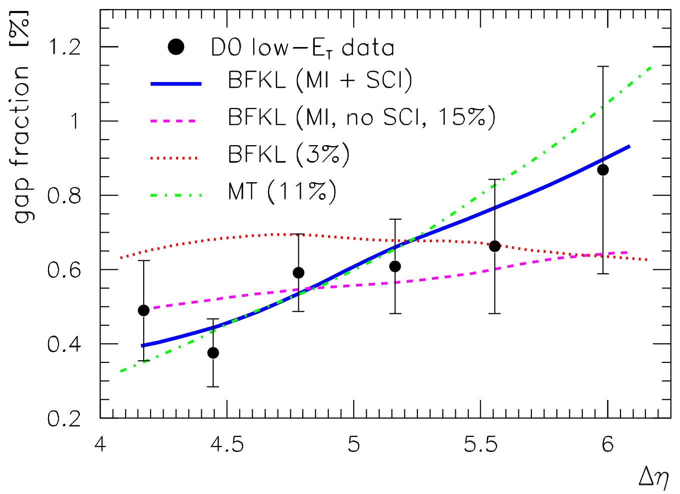
To compare with data, we have to take into account the survival of the gaps, which depends on

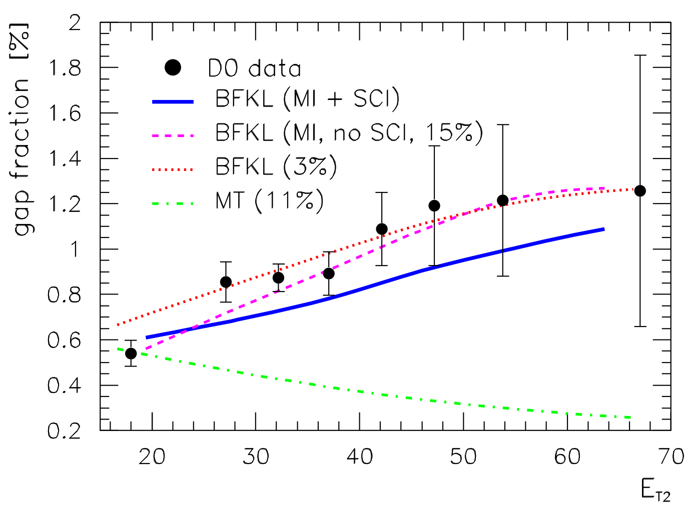
- HO parton emissions
- hadronization
- multiple scatterings
- i.e. the underlying event



- Implemented this as new subprocess in PYTHIA
  `<http://www3.tsl.uu.se/thep/hardcol/>`
- Simulations with multiple interactions and soft colour interactions (which destroy gaps created at parton level)

Comparison to DØ data

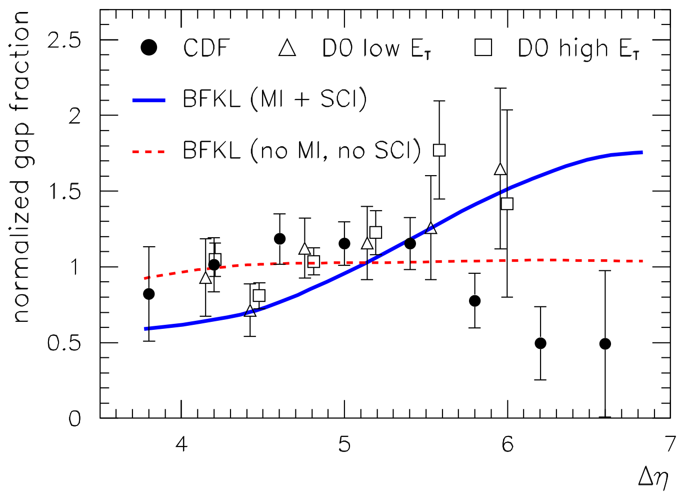


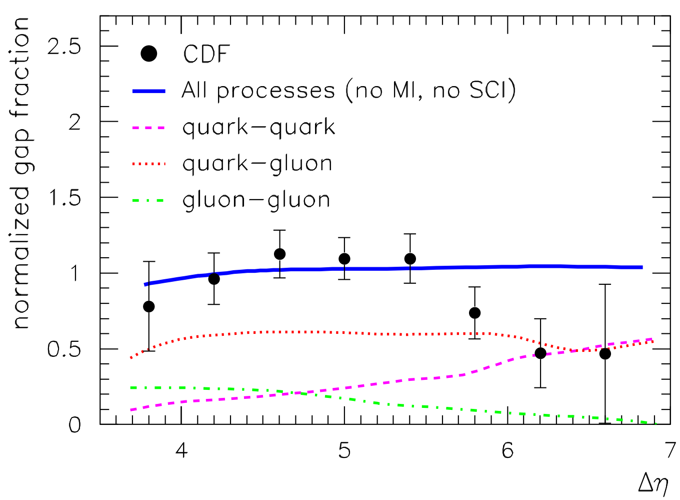


Good agreement with data

- survival probability modelled well
- normalization sensitive to SCI
- SCI makes $\Delta\eta$-shape steeper
- MI without SCI: $\Delta\eta$ flatter but too large survival probability

Comparison to CDF & DØ data

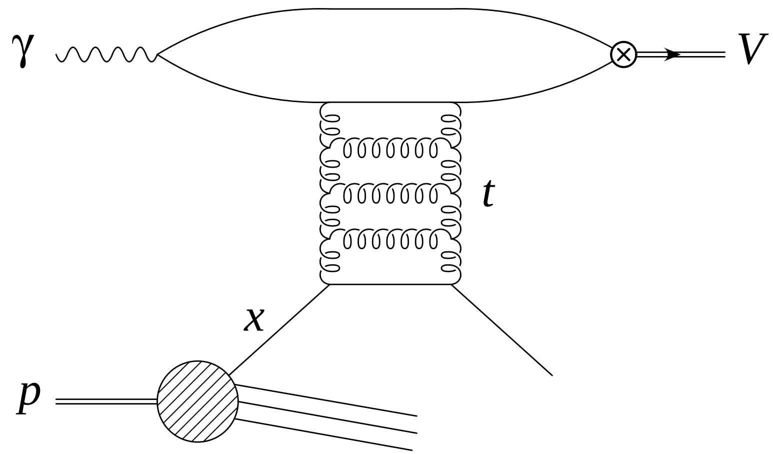




- CDF data *seem* to show a different trend
- SCI suppresses gluon-gluon more than quark-quark $\rightarrow$ rise with y
- Important note: Uncertainties in denominator of the gap fraction might lead to decrease with $\Delta\eta$!
  (QCD uncertainty, not related to colour singlet exchange.)

Vector mesons

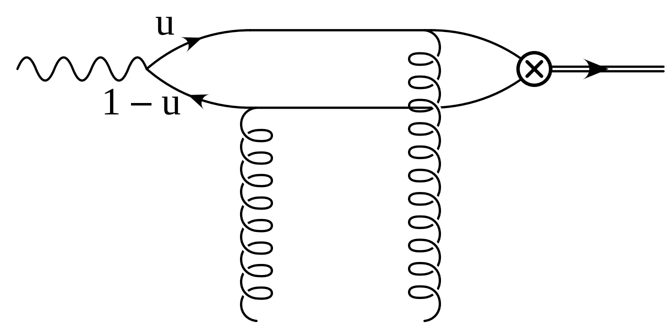
Let's move to vector meson production $\gamma p \to VX$ at large momentum transfer t



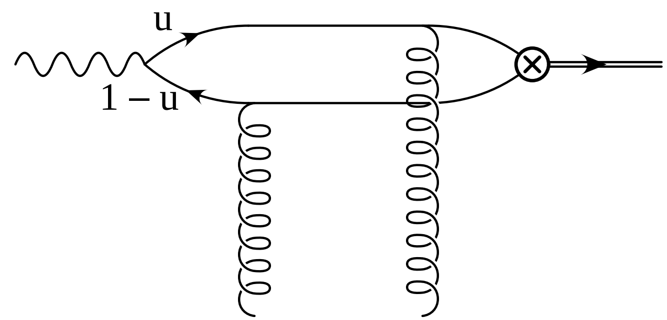
- Experimentally clean
- The large t may serve as a perturbative scale $\to$ possible to test BFKL dynamics ($xW^2 \gg |t| \gg \Lambda_{QCD}$)
- Wave function of vector meson important!

Impact factors & wave functions

Impact factor for $\gamma \to V$ with two attached gluons:



Impact factors & wave functions



Impact factor for $\gamma \to V$ with two attached gluons:

$$\Phi_{\lambda\lambda'}^{\gamma\to V}(\boldsymbol{k},\boldsymbol{q}) = \int du \int d^2\boldsymbol{r}$$

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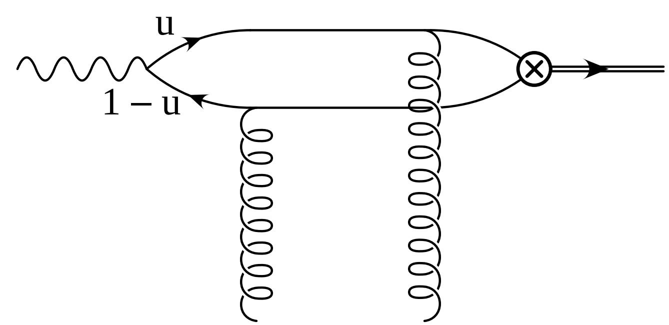

$$\Phi_{\lambda\lambda'}^{\gamma\to V}(\boldsymbol{k},\boldsymbol{q}) = \int du \int d^2\boldsymbol{r}\ [\Psi^*_{V(\lambda')}(u,\boldsymbol{r})]_{\alpha\beta}$$

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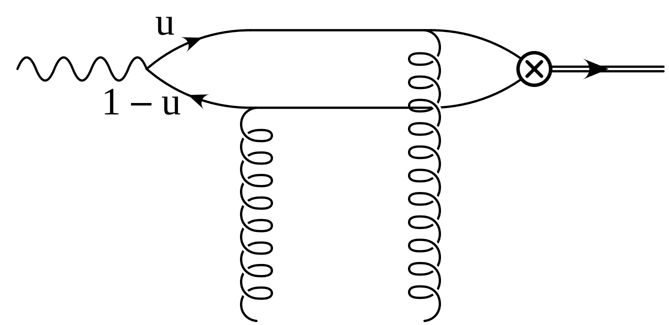

$$\Phi^{\gamma\to V}_{\lambda\lambda'}(\boldsymbol{k},\boldsymbol{q}) = \int du \int d^2\boldsymbol{r}\; [\Psi^*_{V(\lambda')}(u,\boldsymbol{r})]_{\alpha\beta}\, T(\boldsymbol{k},\boldsymbol{q};\boldsymbol{r},u)$$

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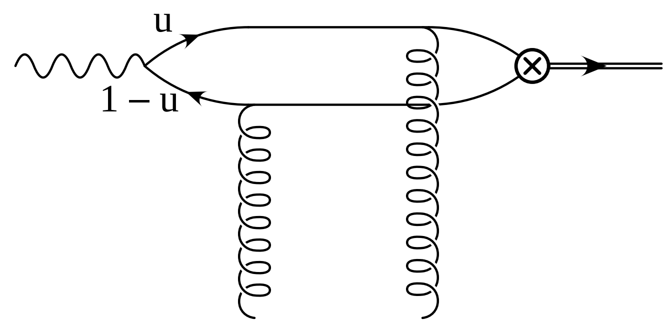

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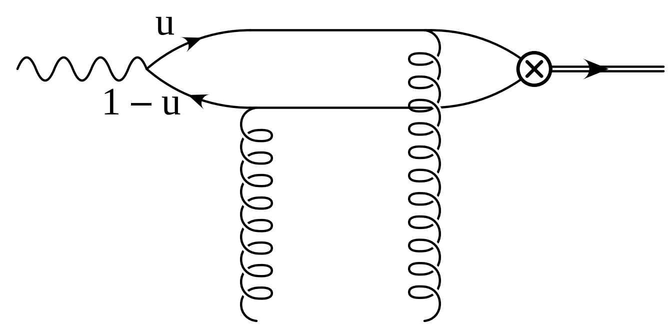

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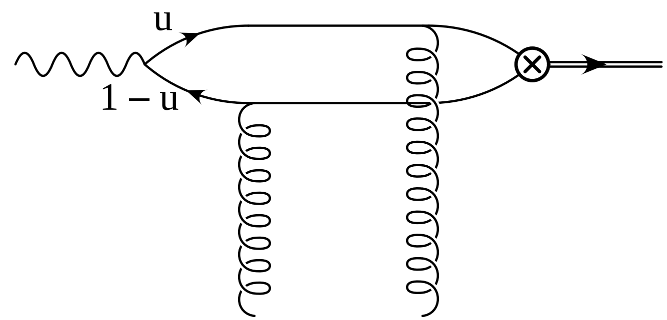

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where $\Psi_{\gamma,V(\lambda)}(u,\boldsymbol{r})$ are the splitting wave functions

$\Rightarrow$ 2-gluon exchange amplitude for $\gamma q \to V q$

$$M_{\lambda\lambda'}(q) = \int d^2\boldsymbol{k} \frac{1}{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2} \Phi^{\gamma V}(\boldsymbol{k},\boldsymbol{q})\, \Phi^{qq}(\boldsymbol{k},\boldsymbol{q})$$

Vector mesons

Three cases:

- Heavy quark limit $\rightarrow$ simpler calculation

  $\Rightarrow$ $J/\psi, \Upsilon$ (and surprisingly, ρ)

  $\Rightarrow$ no polarisation information

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  $\Rightarrow$ allows polarisation study,
  but endpoint divergences make result unreliable

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  $\Rightarrow$ ρ, ϕ, ω

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  but endpoint divergences make result unreliable

- General case: massive quarks

  $\Rightarrow$ $\rho, \phi, \omega, J/\psi, \Upsilon$

  $\Rightarrow$ reliable result, but complicated

The $\gamma \to J/\psi$ impact factor

Non-relativistic approximation (heavy quarks)

- no longitudinal d.o.f., meson wave function $\sim \delta(u - 1/2)$
- $M_V \approx 2m_q$
- Used previously in BFKL calculations in Mueller–Tang ($n = 0$) approximation

  (Forshaw–Ryskin, Bartels–Forshaw–Lotter–Wüsthoff, Forshaw–Poludniowski)

The $\gamma \to J/\psi$ impact factor cont'd

This gives for even n (odd n gives 0)

$$I_{n,\nu}^{\gamma\to V}(q) = \mathcal{C}\,\alpha_s\,\frac{8\pi^2}{|q|^3}\left(\frac{|q|^2}{4}\right)^{i\nu}\left(\frac{\bar{q}}{q}\right)^{n/2}\left(\frac{1}{4}\right)^{|n|/2}\frac{\Gamma(1/2-i\nu+|n|/2)}{\Gamma(1/2+i\nu+|n|/2)}$$

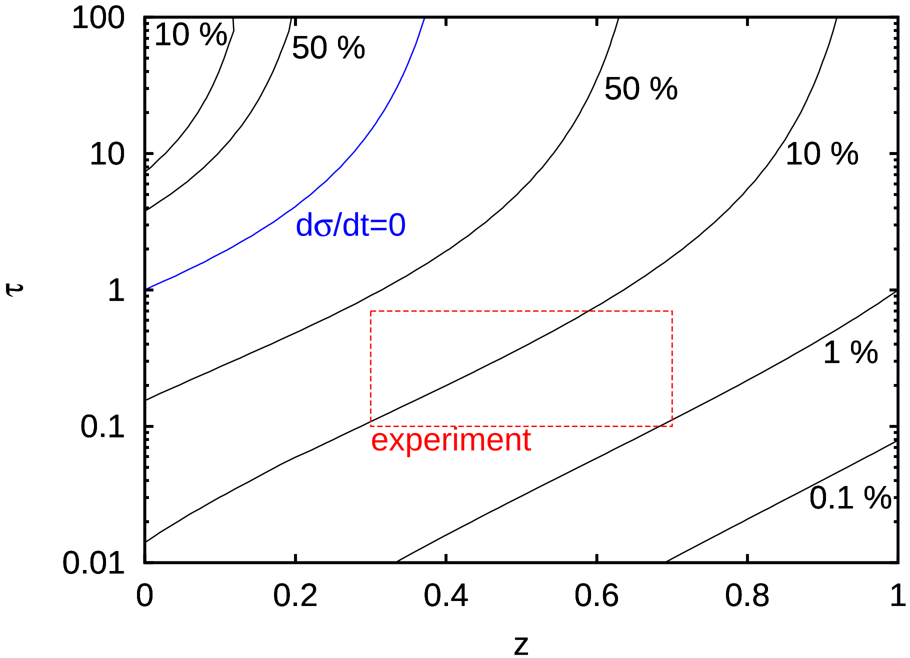
$$\times\int_{-i\infty}^{i\infty}\frac{ds}{2\pi i}\,\tau^{1/2+s+|n|/2}\,\frac{\Gamma(1-s-i\nu)\,\Gamma(1-s+i\nu)}{\Gamma(1-s/2-i\nu/2)\,\Gamma(1-s/2+i\nu/2)}$$

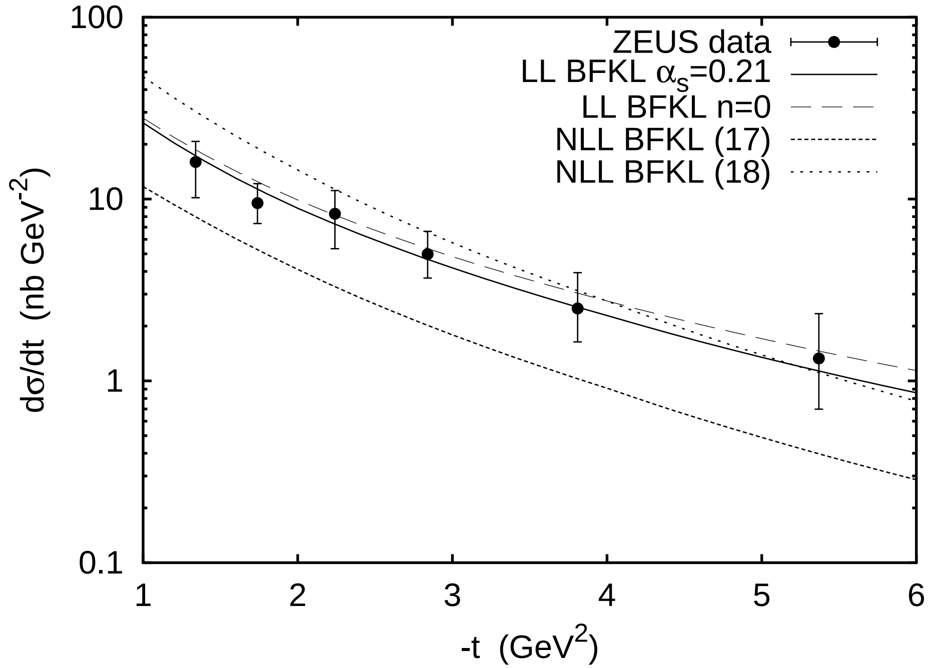
$$\times\frac{\Gamma^2(1/2+s+|n|/2)}{\Gamma(1/2+s/2-i\nu/2+|n|/2)\,\Gamma(1/2+s/2+i\nu/2+|n|/2)}$$

…to plug into the Lipatov solution shown earlier
$\longrightarrow$ gives the parton level amplitude

J/ψ results

[EPJ C26 (2002) 219, with L. Motyka & G. Poludniowski]





$$\mathcal{E} = \frac{|A_{exact}|^2 - |A_0|^2}{|A_0|^2} \qquad \frac{d\sigma}{dt}$$

$(\tau = \frac{|t|}{M_V^2}, \quad z = \frac{3\alpha_s}{2\pi}\, y, \quad y =$ rapidity)

J/ψ: influence of higher conformal spins ($n \neq 0$) not so large at present experiments!

See also upcoming H1 paper. . .

The general $\gamma \to V$ impact factor

Ivanov, Kirschner, Schäfer & Szymanowski [PLB 478 (2000) 101] give the 2-gluon exchange amplitudes $M_{(++)}$, $M_{(+-)}$, $M_{(+0)}$ for $\gamma q \to V q$ (light mesons only, assumes massless quarks)

- $M_{(++)}$: $\gamma_T \to V_T$
- $M_{(+-)}$: $\gamma_T \to V_L$
- $M_{(+0)}$: $\gamma_T \to V_T$

Experimentally: $M_{(++)}$ and $M_{(+-)}$ cannot be distinguished

$\to$ measure spin density matrix elements (parametrisations of meson decay products)

For instance:

$$M_{+0}^{\text{even}} = -iC_V \alpha_s^2 \int \frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2(\boldsymbol{k}-\boldsymbol{q})^2} \frac{d^2\boldsymbol{r}\,du}{4\pi}\, m\, K_1(m|\boldsymbol{r}|) f^{\text{dipole}} \frac{\boldsymbol{e}^+ \cdot \boldsymbol{r}}{|\boldsymbol{r}|} f_V\, (1-2u)\phi_\parallel(u)$$

For instance:

$$M_{+0}^{\text{even}} = -iC_V\alpha_s^2 \int \frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2(\boldsymbol{k}-\boldsymbol{q})^2} \underbrace{\frac{d^2\boldsymbol{r}\,du}{4\pi}\, m\,K_1(m|\boldsymbol{r}|)\, f^{\text{dipole}}\, \frac{\boldsymbol{e}^+\cdot\boldsymbol{r}}{|\boldsymbol{r}|}\, f_V\,(1-2u)\phi_\parallel(u)}_{\sim\,\Phi^{q\to q}\,\times\,\Phi^{\gamma\to V}_{(+0)}\,\times\,\delta(\boldsymbol{k}-\boldsymbol{k}')}$$

For instance:

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$$\underbrace{\frac{d^2\boldsymbol{r}\,du}{4\pi}\, m\, K_1(m|\boldsymbol{r}|)\, f^{\text{dipole}}\, \frac{\boldsymbol{e}^+ \cdot \boldsymbol{r}}{|\boldsymbol{r}|}\, f_V\, (1-2u)\phi_{\|}(u)}_{\sim\, \Phi^{q\to q}\, \times\, \Phi^{\gamma\to V}_{(+0)}\, \times\, \delta(\boldsymbol{k}-\boldsymbol{k}')}$$

- $\phi_{\|}(u) = 6u(1-u)$ is the VM distribution amplitude

$\Rightarrow$ Pick the $\Phi^{\gamma\to V}_{(hel.)}$ from 2-gluon formula!

First with massless quarks ($m \to 0$), then keeping finite m

We have done the calculation for

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- zero quark mass $\longrightarrow$ light meson result

  [proc. DIS 2002 with L. Motyka & G. Poludniowski]

  - endpoint divergences ($u \to 0, 1$) $\Rightarrow$ arbitrary cut-off

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- zero quark mass $\rightarrow$ light meson result

  [proc. DIS 2002 with L. Motyka & G. Poludniowski]

  - endpoint divergences ($u \rightarrow 0, 1$) $\Rightarrow$ arbitrary cut-off

- finite quark mass $\rightarrow$ general result

  [TSL/ISV-2003-0269 with J. Forshaw, L. Motyka & G. Poludniowski]

  - no end-point divergence
  - yields both light & heavy meson results in appropriate limits

General formula

The amplitudes are given by ($\lambda, \lambda' \alpha, \beta, a$ determine which helicity amplitude)

$$M_{\lambda\lambda'} = \mathcal{C} \int_0^1 du \; F_{\lambda\lambda'}(u)$$

$$\times \sum_n \int d\nu \frac{\nu^2+n^2}{[\nu^2+(n-1/2)^2][\nu^2+(n+1/2)^2]} \frac{\exp[\chi_{2n}(\nu) z]}{\sin(i\pi\nu)} \; I_{\alpha\beta}(\nu, 2n, q, u; a),$$

where the integral $I_{\alpha\beta}$ is

$$\begin{aligned} I_{\alpha\beta}(\nu,n,q,u;a) \quad &= \quad -\frac{m}{2} \int_{C'-i\infty}^{C'+i\infty} \frac{d\zeta}{2\pi i} \Gamma(a/2-\zeta)\Gamma(-a/2-\zeta) \, \tau_q^\zeta \; (i \, \mathrm{sign} \, (2u-1))^{\alpha-\beta+n} \\ &\times \quad \left(\frac{4}{|q|}\right)^4 [\sin\pi(\alpha+\mu+\zeta) \;\; B(\alpha,\mu,q^*,u,\zeta) \, B(\beta,\widetilde{\mu},q,u^*,\zeta) \\ &- \quad (-1)^n \sin\pi(\alpha-\mu+\zeta) \;\; B(\alpha,-\mu,q^*,u,\zeta) \, B(\beta,-\widetilde{\mu},q,u^*,\zeta) \,] \end{aligned}$$

and the conformal blocks $B(\alpha, \mu, q^*, u, \zeta)$ are defined:

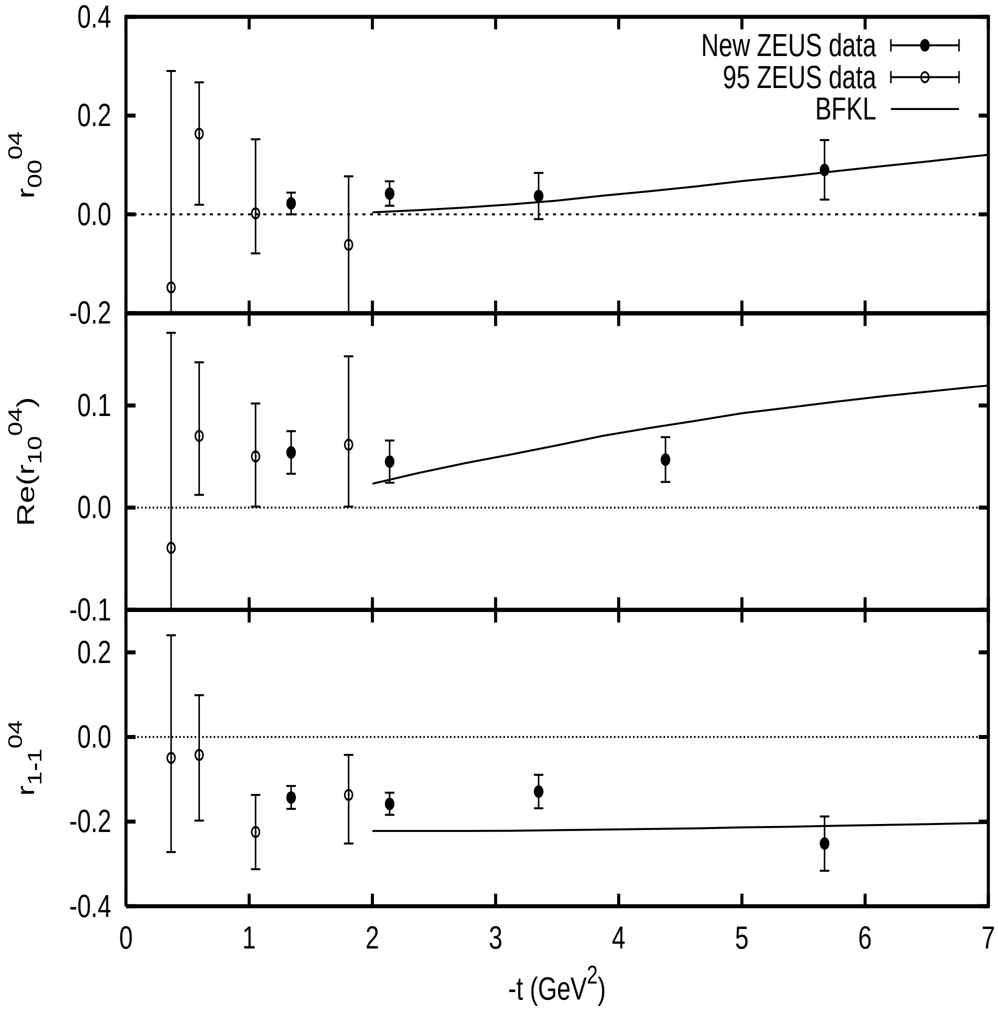
$$B(\alpha, \mu, q^*, u, \zeta) = (-4u\bar{u})^{-(\mu+2+\alpha+\zeta)/2} \left(\frac{4}{q^*}\right)^\alpha 2^{-\mu} \, \frac{\Gamma(\mu+2+\alpha+\zeta)}{\Gamma(\mu+1)}$$

$$\,_2F_1\left(\frac{\mu+2+\alpha+\zeta}{2}\, , \; \frac{\mu-1-\alpha-\zeta}{2} \, ; \; \mu+1 \, ; \; \frac{1}{4u\bar{u}}\right).$$

ρ results

Data: ρ, ϕ show helicity flip, J/ψ don't

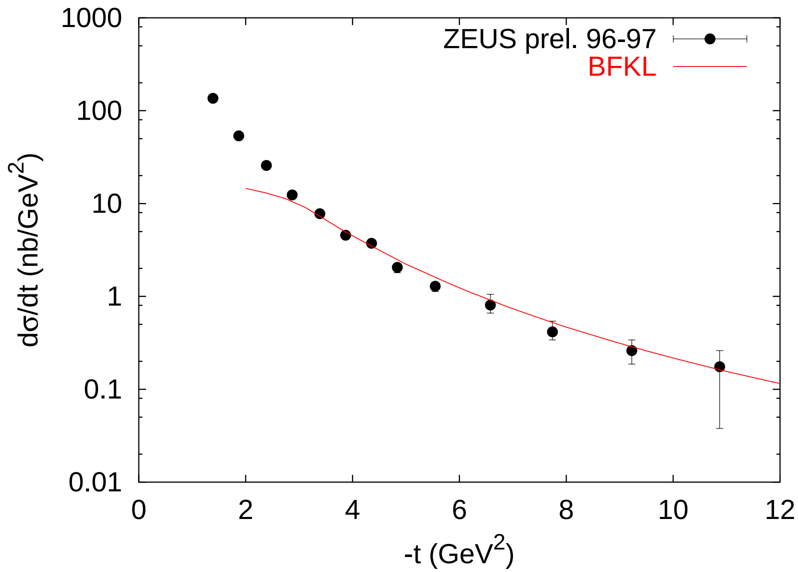
Spin density matrix elements: (all zero if there is no helicity flip)



The light meson calculation shows general agreement

- Many conformal spins needed
- Spin double-flip dominates
- Agreement real?
- $m_q > 0$ formulas derived, comparison underway...

ρ results



- Reproduces $d\sigma/dt$ with chosen parameters
- Small-$|t|$ dip because of cut-off in u integration: $u_{\min} = -m_\rho^2/t$ (too restrictive, taken from Ivanov et al)

Complete analysis including quark masses and chiral-odd contributions is on the way!

Note that light meson data does not agree with fixed-order calculation!

Is this the first clear BFKL signature??

Conclusions

- Hard colour singlet exchange is described by BFKL
- Non-zero conformal spins needed for y not very large
- Gaps between jets:
  - Mueller-Tang doesn't work
  - Full numerical solution of BFKL with MC simulation (underlying event) reproduces data
- Vector meson production at large $|t|$:
  - We have obtained exact LL BFKL amplitudes
  - J/ψ: Higher conformal spins $\sim$ 10% effect at HERA
  - Prel. comparison with data looks good
- Increased understanding and evidence for BFKL