A model for the non-perturbative x-shape of parton distributions in hadrons

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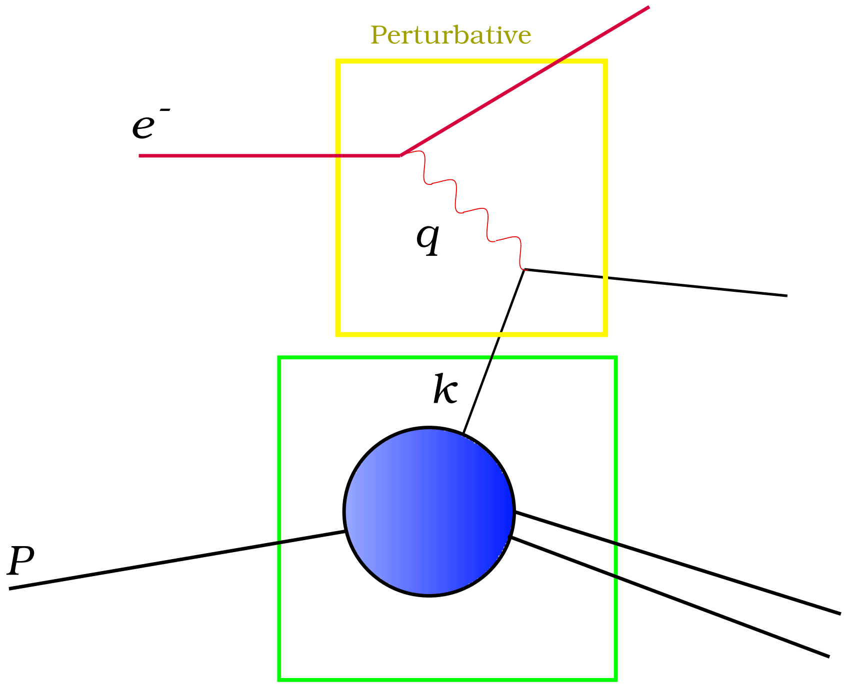
Introduction to pdfs

The cross-section for hadronic interactions can be factorized into a perturbative part and a non-perturbative part.



$$\frac{d\sigma}{dk dQ^2}(\gamma p \to A) = \sum_i f_i(k, Q^2) \frac{d\sigma}{dQ^2}(\gamma q_i \to q_i)$$

$d\sigma(\gamma q_i \to q_i)$: calculated perturbatively using Feynman diagrams

$f_i(k, Q^2)$: probability of finding quark q_i with momentum k in proton, $Q^2 = -q^2$.

Instead of k, usually x ("Bjorken-x") is used, which is defined such that in the "infinite-momentum" frame $k = xP$.

Introduction to pdfs, contd.

The $f_i(x, Q^2)$s are called *parton distribution functions*. They cannot be calculated from first principles, but must be supplied in some other way.

The dependence of $f_i(x, Q^2)$ on Q^2 can be found using the *Altarelli-Parisi* equation, but

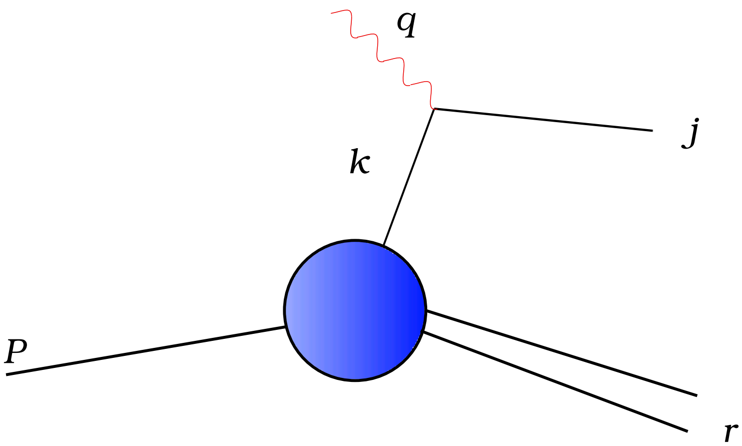
we need starting distributions at some scale Q_0^2

These starting distributions are usually parametrizations of some arbitrary functions based on DIS results.

We have constructed a physical model to get starting distributions.

The Edin-Ingelman model

In the rest frame of the hadron we assume Gaussian fluctuations ($k^\mu \in N(0, \sigma_i)$) in the momenta of partons and add kinematic constraints:

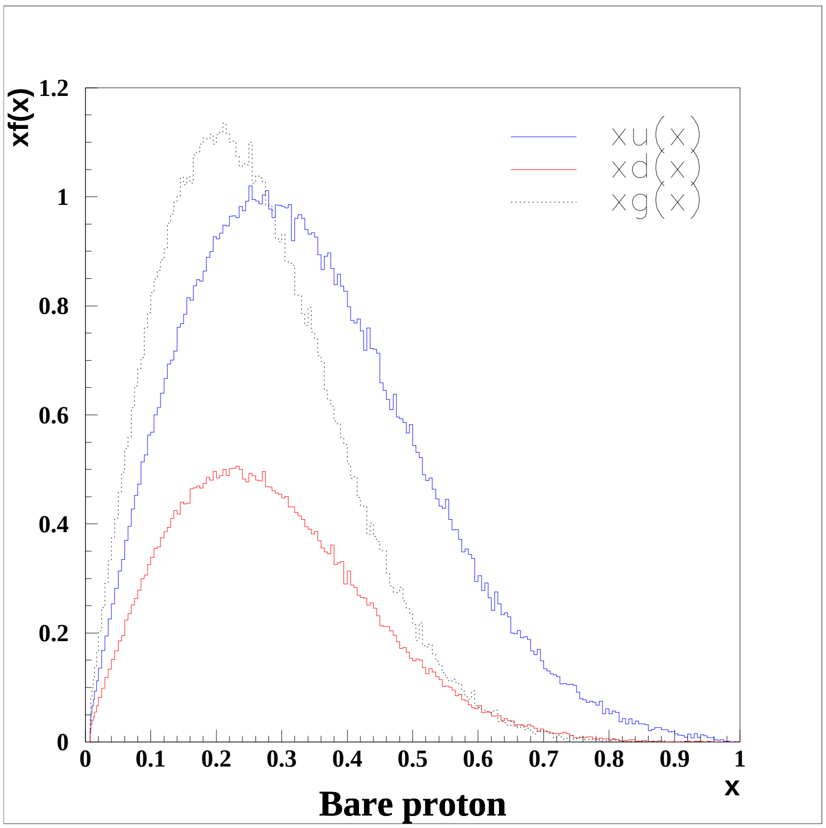


$$0 < j^2 < W^2$$

$$r^2 > 0$$

where $W^2 = (P + q)^2$.

Monte Carlo-simulating using $x = \frac{k_+}{P_+}$ gives



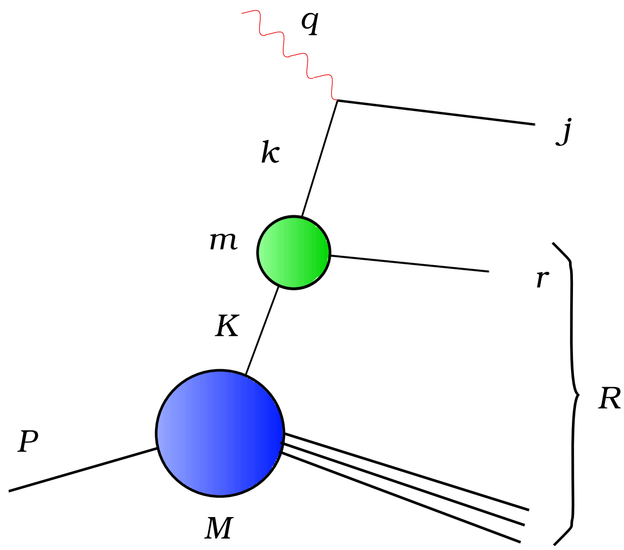
Hadronic fluctuations

In the basic model only the valence distributions are simulated. To get sea partons as well we assume hadronic fluctuations. This means that e.g. the proton is viewed as consisting of a sum of several quantum states,

$$
\begin{aligned}
|p\rangle &= \alpha_0|p_0\rangle + \alpha_{p\pi^0}|p_0\pi^0\rangle + \alpha_{n\pi^+}|n\pi^+\rangle + \ldots + \\
&\quad + \alpha_{\Lambda K}|\Lambda K^+\rangle + \ldots
\end{aligned}
$$

The fluctuations are also assumed to have a Gaussian momentum distribution in the rest frame of the proton and the energy is fluctuated around the resulting kinetic energy. The photon (probe) is then interacting with a parton in the hadronic fluctuation.

Hadronic fluctuations, contd.



$$x_F = \frac{K_+}{(K+K_{\text{partner}})_+}$$

$$x_i = \frac{k_+}{K_+}$$

$$x = x_F \cdot x_i$$

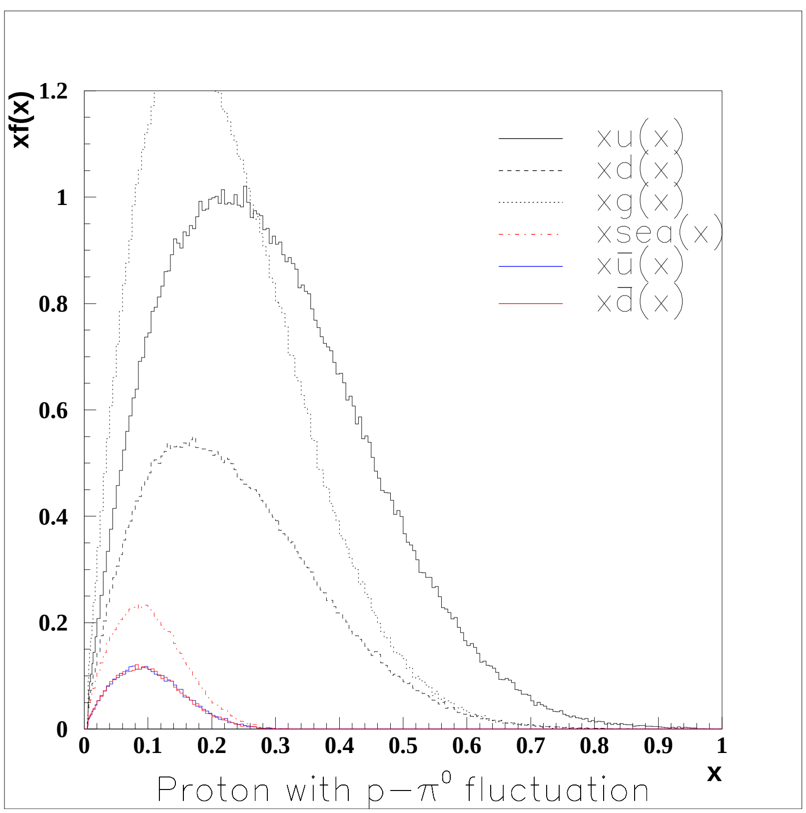
Kinematic constraints:

$$0 < j^2 < x_F W^2$$

$$r^2 > 0,\ R^2 > 0$$

where F stands for "Fluctuation".

The resulting sea distribution using the dominant $|p_0\pi^0\rangle$ fluctuation:



Ratio between fluctuations

Using old fashioned perturbation theory,

$$P(\text{fluctuation}) \propto \Delta E^{-2}$$

Letting $\alpha^2 = \sum_F \alpha_F^2$ (referring to the fluctuation expansion of the proton), we have for the dominant fluctuations $|p_0\pi^0\rangle$ and $|n\pi^+\rangle$ that

$$\alpha_{p\pi^0}^2 \approx \tfrac{1}{2}\alpha^2$$
$$\alpha_{n\pi^+}^2 \approx \tfrac{1}{2}\alpha^2$$

while for the first strange fluctuation $|\Lambda K^+\rangle$ we get

$$\alpha_{\Lambda K}^2 \approx \left(\frac{m_{\pi^0}}{m_\Lambda + m_{K^+} - m_p}\right)^2 \alpha_{p\pi^0}^2 \approx 0.02\,\alpha^2$$

(assuming that the coupling between the proton and all fluctuations is the same).

We get α^2 by fitting to HERA F_2 data using MINUIT.

$\gamma \to$ Vector meson component

As $Q^2 \to 0$ photons may fluctuate to vector mesons,

$$|\gamma\rangle = C_0|\gamma_0\rangle + \sum_{V=\rho^0,\omega,\phi} \frac{e}{f_V}|V\rangle$$

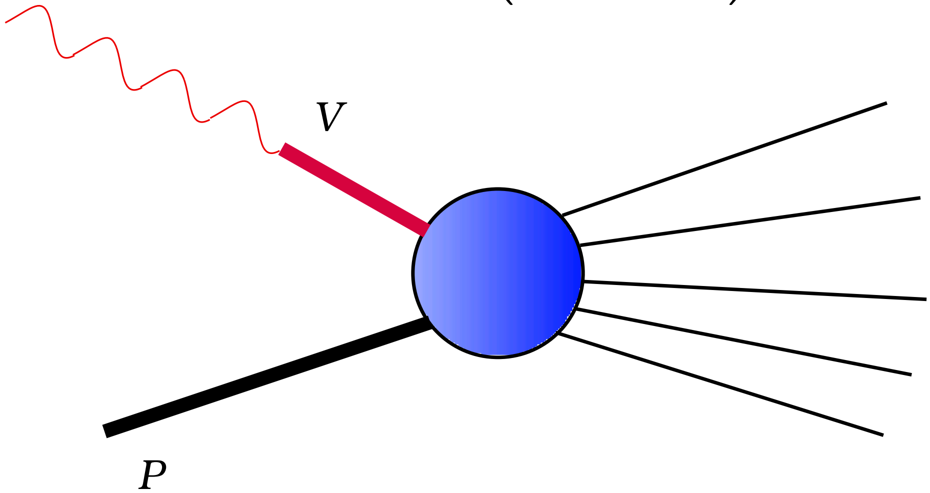
Generalizing to $Q^2 \neq 0$ we get

$$\frac{d\sigma}{dxdQ^2} = \frac{\alpha}{2\pi}\frac{1+(1-y)^2}{xQ^2}\frac{4\pi\alpha}{f_\rho^2}\left(\frac{m_\rho^2}{m_\rho^2+Q^2}\right)^2 \sigma_{\rho p}(s_{\gamma p})$$

where $\sigma_{\rho p}(s_{\gamma p}) = As_{\gamma p}^{\epsilon} = AQ^{2\epsilon}x^{-\epsilon}$ and $\epsilon = 0.08$ from Regge fits to data.

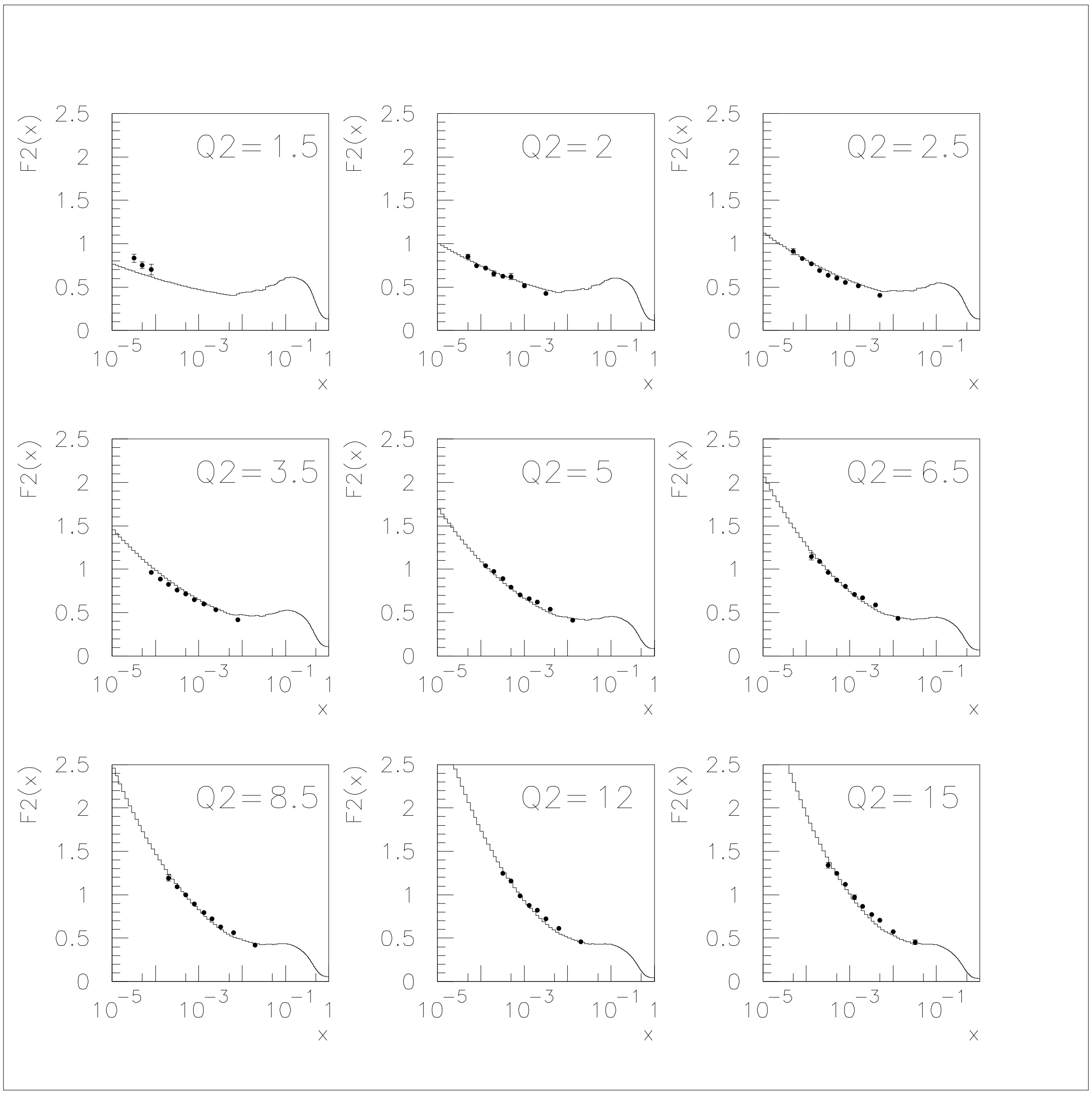
This gives a vector meson dominance (VMD) model contribution to the DIS cross-section (A is fitted to HERA data):

$$F_2^{\rm VMD}(x,Q^2) = \frac{A}{\pi f_\rho^2}\left(\frac{m_\rho^2}{m_\rho^2+Q^2}\right)^2 Q^{2(1+\epsilon)}x^{-\epsilon}$$

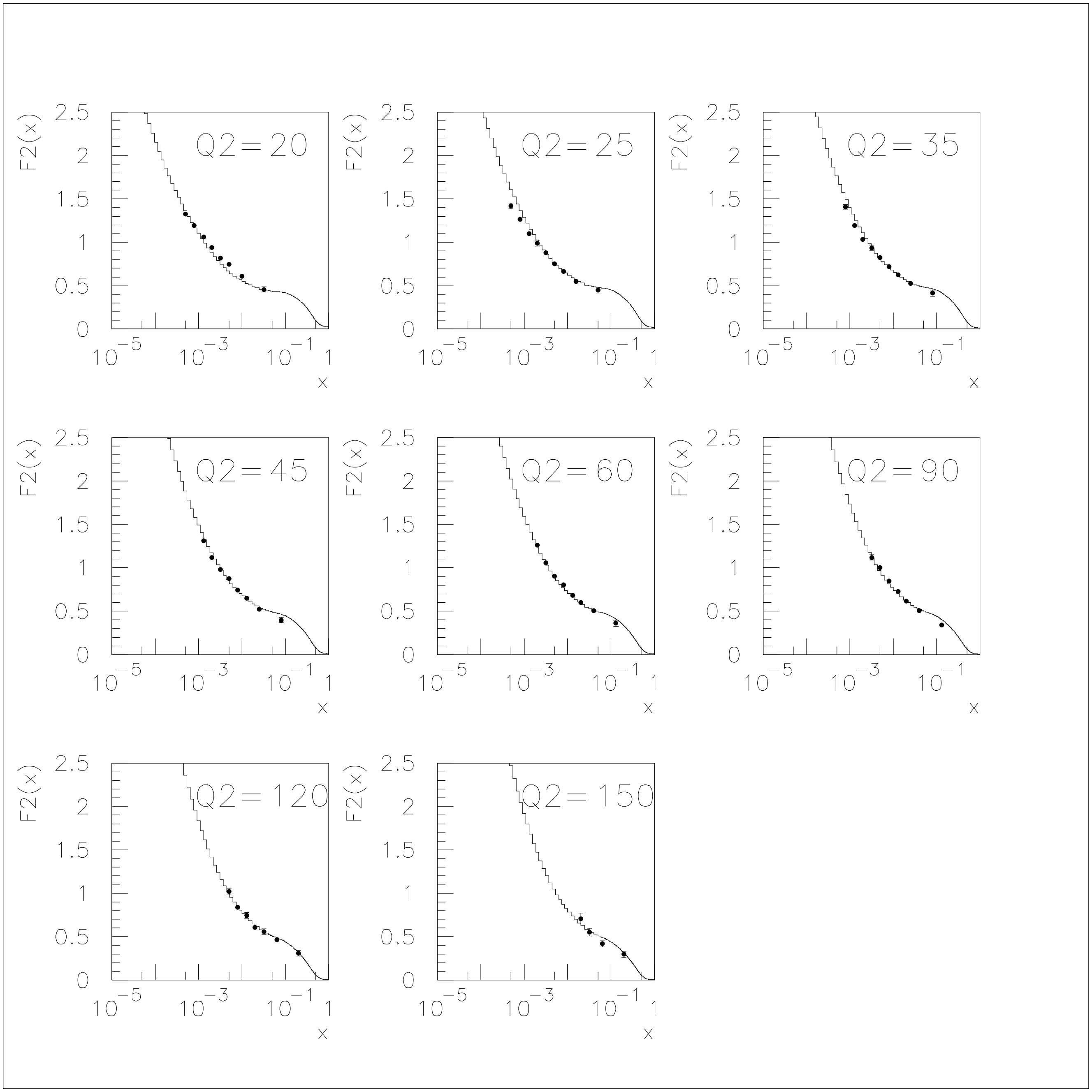


Fit to HERA F_2 data

$Q_0^2 = 0.63\text{GeV}^2$,
$\sigma_u = 180\text{MeV}$, $\sigma_d = 150\text{MeV}$, $\sigma_g = 135\text{MeV}$,
$\sigma_F = 58\text{MeV}$, $\alpha^2 = 0.0933$,
$\frac{A}{\pi f_\rho^2} = 1.05$

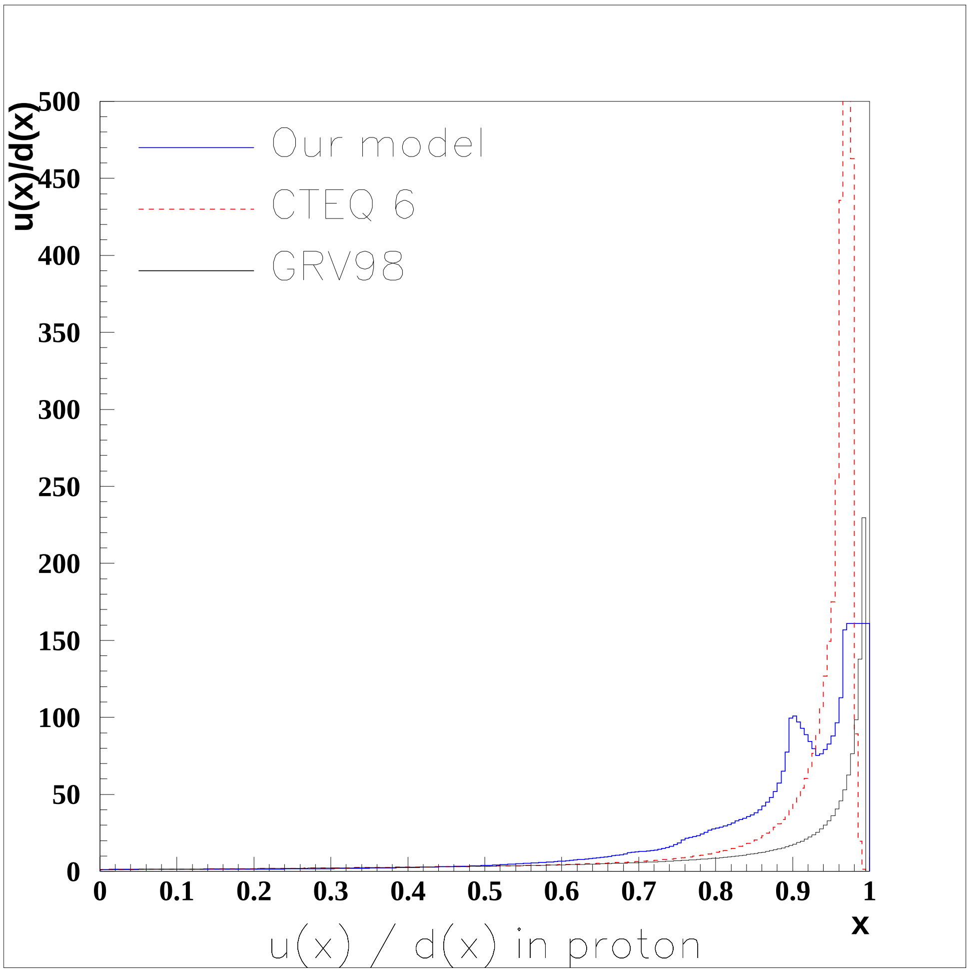


Fits to HERA data, contd.



Results 1: $u(x) - d(x)$ difference

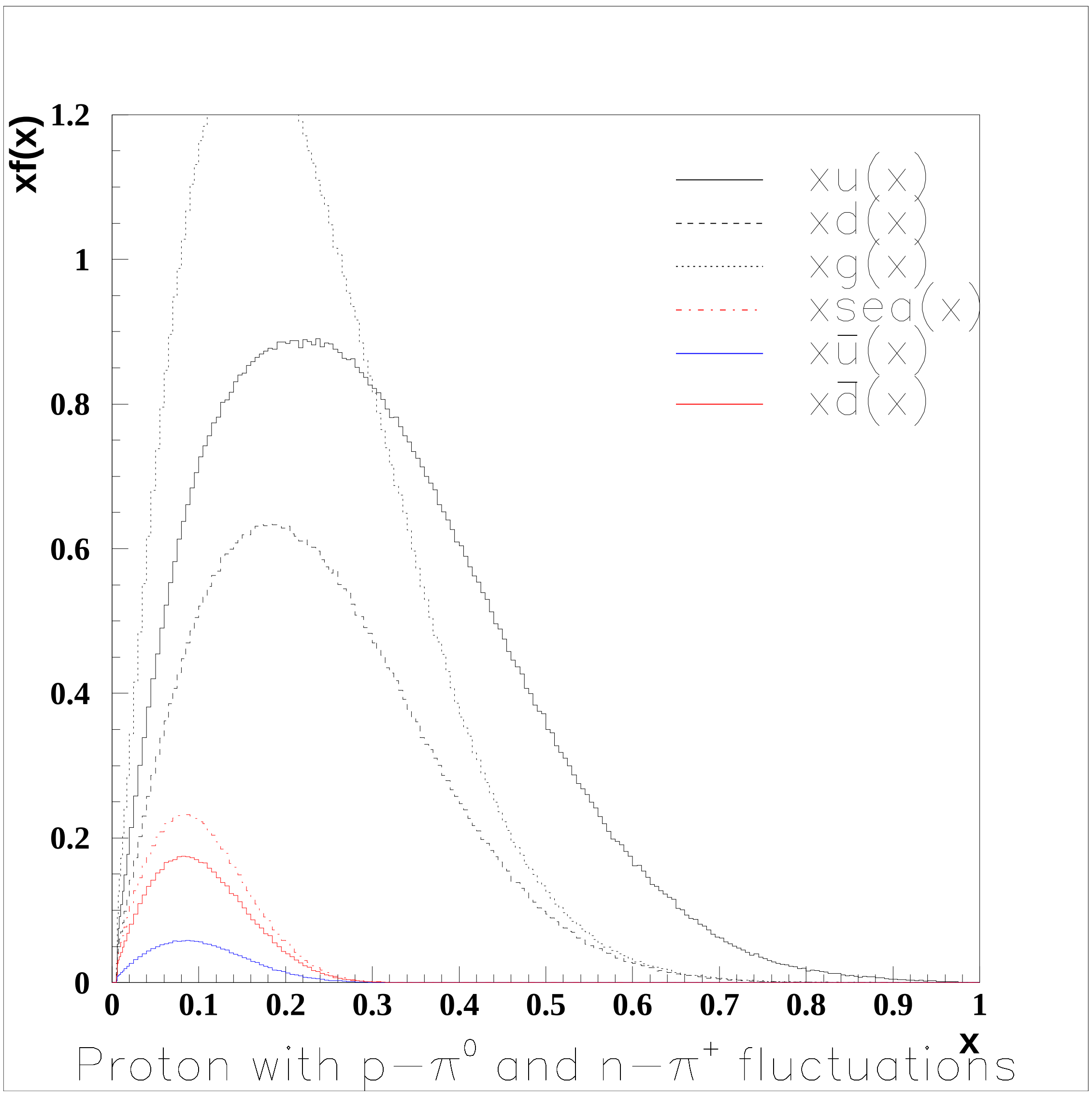
Due to the fluctuation $|n\pi^+\rangle$ we automatically get a difference between the slopes of $u(x)$ and $d(x)$ for high x predicted by data and found in the standard parametrizations:



This is because the neutron is at lower x due to momentum taken by the π-meson, giving a suppression of $d(x)$ relative to $u(x)$ for $x \to 1$.

Results 2: $\bar{u}(x) - \bar{d}(x)$ difference

We also get a difference between $\bar{u}(x)$ and $\bar{d}(x)$ at lower x when using both $|p\pi^0\rangle$ and $|n\pi^+\rangle$ fluctuations:



This agrees qualitatively with fitted pdfs.

The NuTeV anomaly

NuTeV is an experiment measuring the ratio of neutral to charged current interactions in $\nu - N$ scattering. This provides a value of the Standard Model $\sin^2\theta_W$ which differs from previous measurements in other interactions by 3σ.

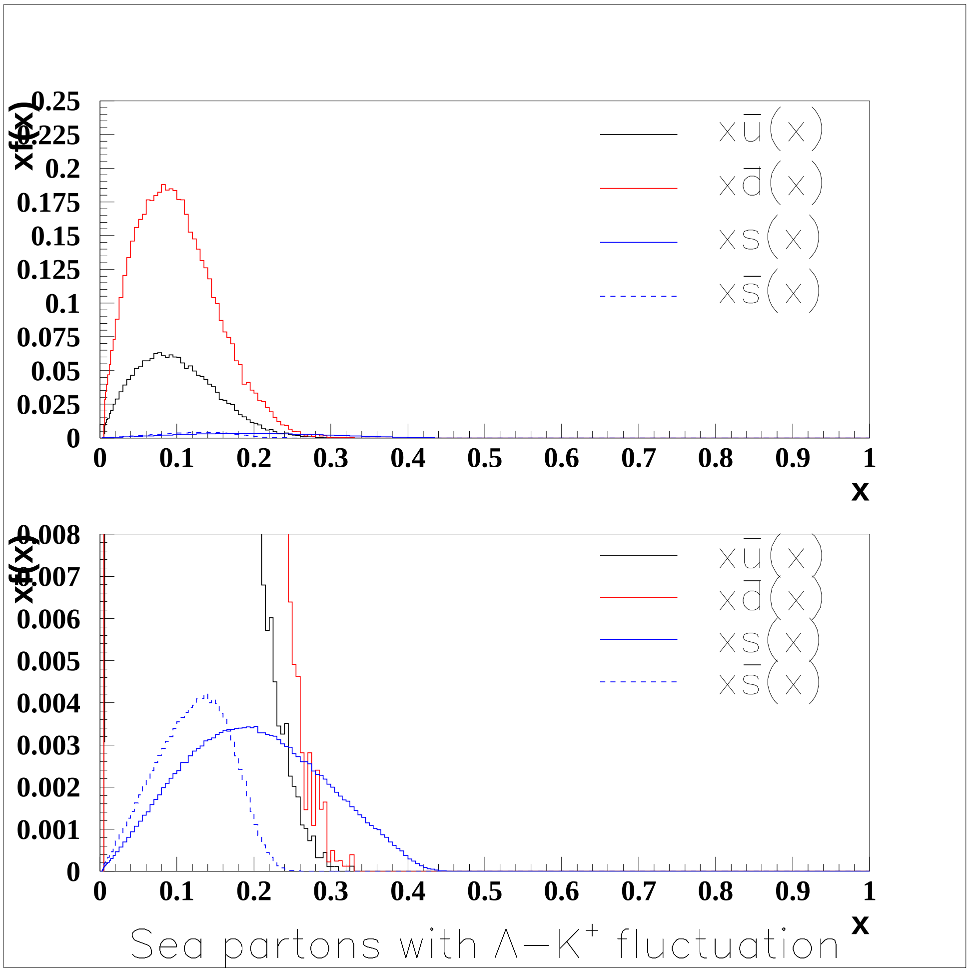
Davidson et al. have contributed a list of possible explanations to the anomaly, both within the SM and "new physics" explanations. One of the explanations proposed within the SM is that $s(x)$ and $\bar{s}(x)$ in the proton are different in shape;

$$\int_0^1 [s(x) - \bar{s}(x)]dx = 0 \quad \text{but}$$
$$s^- = \int_0^1 [xs(x) - x\bar{s}(x)]dx \neq 0$$

An $s^- \approx 0.002$ would reduce the discrepancy between the NuTeV result and the SM prediction to 1.5σ.

Results 3: $s - \bar{s}$ difference

The dominant strange fluctuation is $|\Lambda K^{+}\rangle$, giving an asymmetry between s and $\bar{s}$, since the larger mass Λ gives an s at higher x then the $\bar{s}$ in K^{+}:



The resulting difference (at Q_0^2):

$$s^{-} = \int_0^1 [xs(x) - x\bar{s}(x)]dx \approx 0.0003$$

Conclusions

- We have a physical model for the x-shape of parton distribution functions in hadrons.
- At low Q^2 photon fluctuations to vector mesons are needed to fit the cross-section.
- The model gives a difference between the u and d distributions for $x \to 1$ in qualitative agreement with fitted parametrizations.
- We also get a difference between $\bar{u}$ and $\bar{d}$ at lower x, in qualitative agreement with fitted parametrizations.
- The $s - \bar{s}$ difference predicted by the model does not suffice to explain the NuTeV anomaly.