Energy dependent coupling constants
– a quantum mechanical reality

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Outline

- Introduction
- Why couplings depend on energy and how to observe it
- Making calculations with a running coupling
- Outlook and conclusions

Running coupling constants

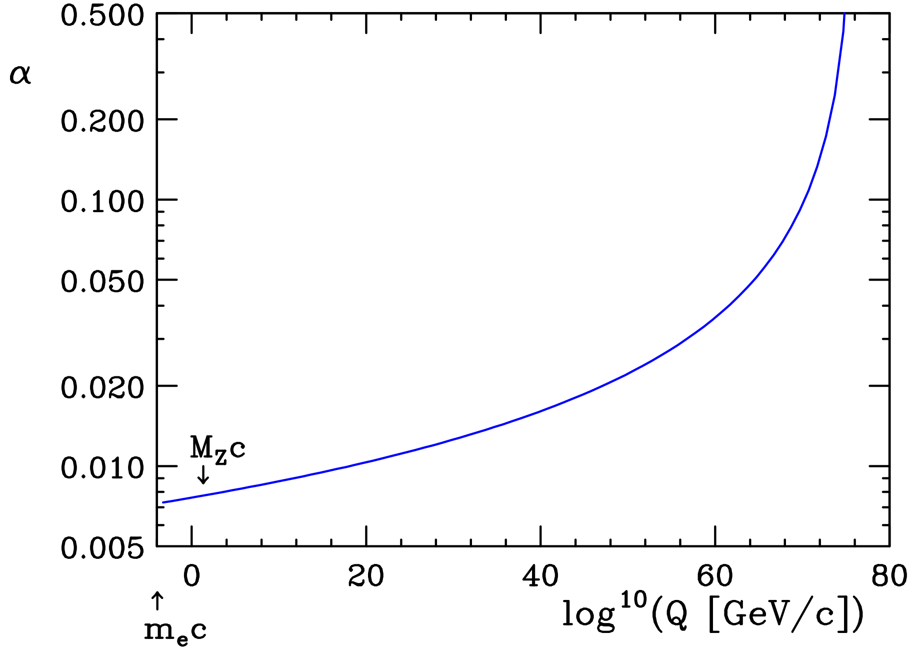
Fine structure constant: $\alpha = \dfrac{e^2}{4\pi\epsilon_0\hbar c} = \dfrac{1}{137.035\,999\,76\,(50)} = 0.007\,297\,352\,533\,(27)$

But, measurements at high energies (LEP at CERN): $\alpha \simeq \dfrac{1}{128} \simeq 0.00781$

(in agreement with theory)

What is going on? α is *not* constant but "energy"-dependent (more precisely momentum-transfer Q [E/c])

Extrapolating to higher energies using quantum electrodynamis (QED) α becomes infinitely large – "the Landau pole"



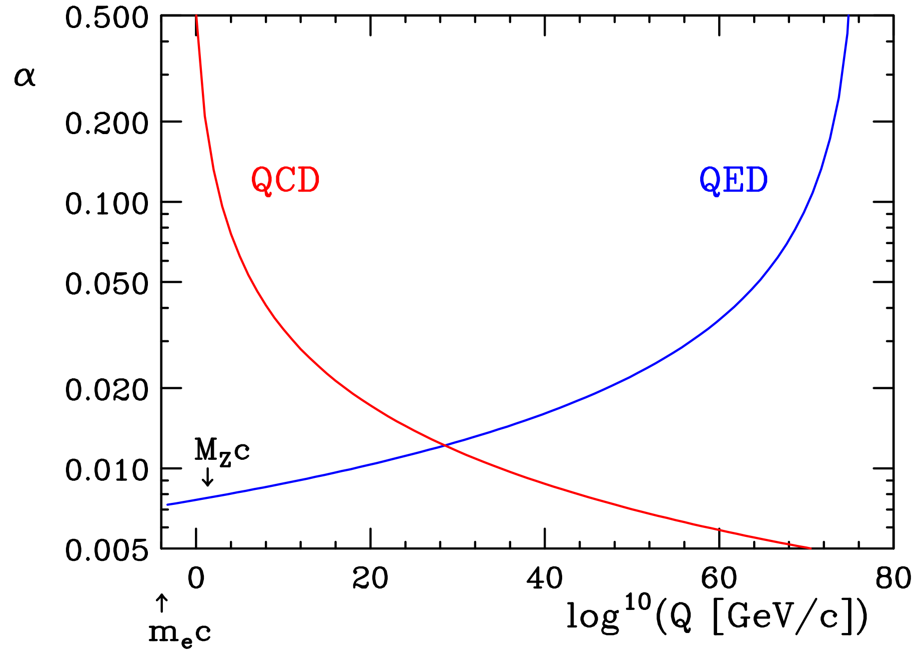
Why is the fine structure constant (coupling) behaving this way?

What about other interactions, e.g. the strong interaction?

In the sixties Gell-Mann and Zweig showed how hadrons (e.g. protons and neutrons) can be understood as composite states of quarks – are quarks real or mathematical?

1. As the name implies the strong interaction is strong – the coupling is large at small energies

2. To be able to describe the strong interaction using quarks the coupling has to be small



1973: Gross & Wilczek and Politzer ('t Hooft) showed that the strong coupling decreases as the energy is increased (birth of quantum chromodynamics (QCD), the theory of strong interactions)

⇒ quarks are real (not mathematical), possible to make precise calculations also in strong interactions at large momentum-transfers (asymptotic freedom)

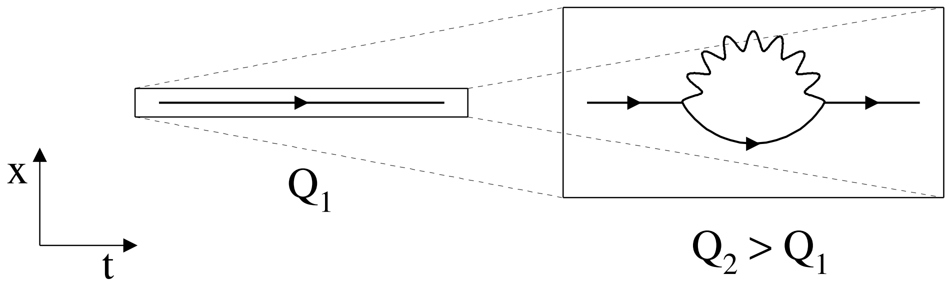
Quantum electrodynamics (QED)

Describes the interactions of electrically charged particles mediated by photons

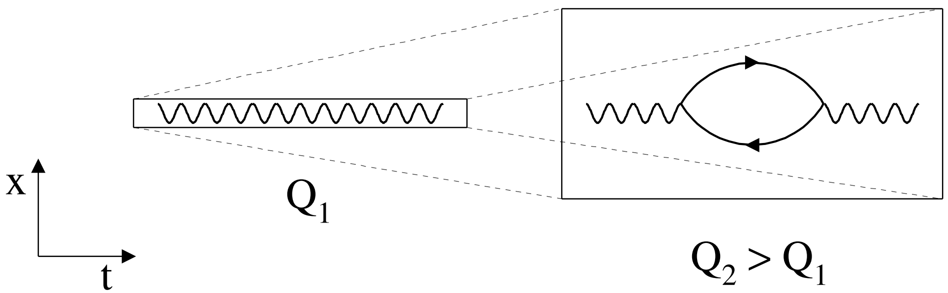
Main differences to classical electrodynamics: uncertainty principle $\Delta x \Delta p \geq \hbar$ and quantisation of fields (photons) $\Rightarrow$ the number of particles is *not* constant

Increasing the resolution $Q \sim \hbar / \Delta x$ we resolve quantum fluctuations

electron:

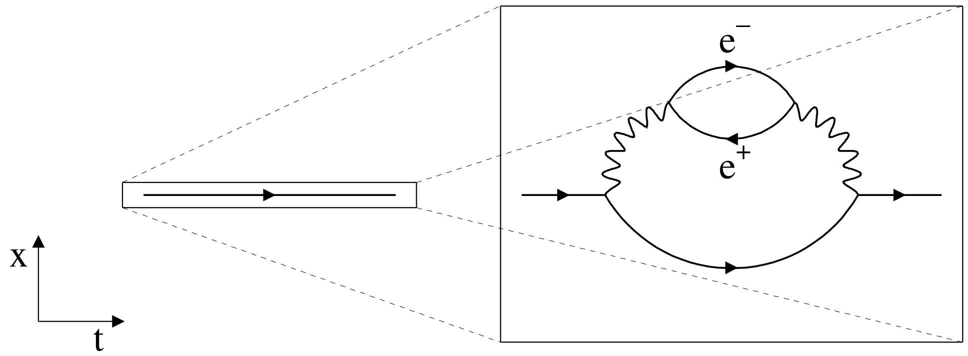


photon:



Vacuum polarisation in QED

Combining these two types of fluctuations will polarise the vacuum around an electron

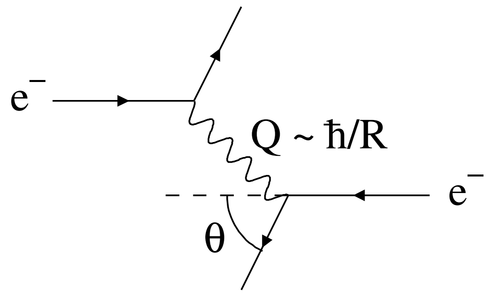


Increasing the resolution $Q \sim \hbar/\Delta x \Rightarrow$ what we thought was one charge is actually fluctuating into several

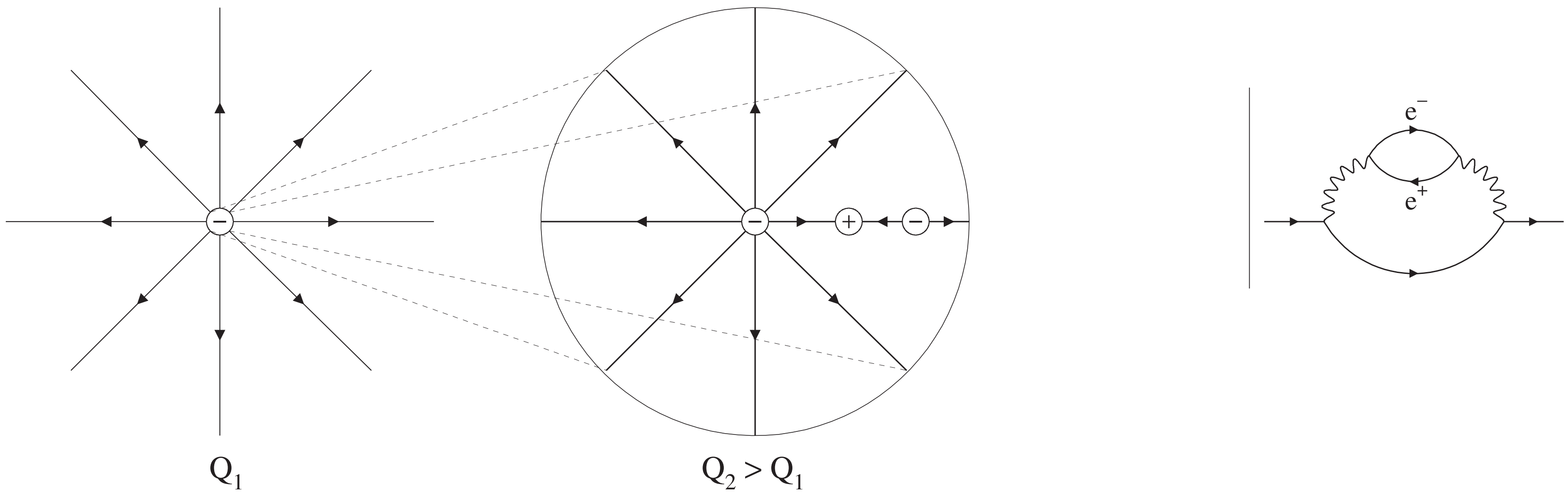
How can this be observed? (Why should we care about quantum fluctuations?)

to probe short distances $R \sim \Delta x$ we need processes with large momentum-transfers $Q \sim \hbar/R$

for example by scattering two high energy electrons to large angle ($Q = 2E \sin\frac{\theta}{2}/c$)



The electric field (at distance $R \sim \hbar/Q$) from an electron in vacuum: $|\vec{E}| = \dfrac{e(R)}{4\pi\varepsilon_0 R^2}$



Quantum fluctuations polarise the vacuum and *screen* the charge at large distances cf. dielectric medium with $\varepsilon(R) > \varepsilon_0$, $\left(|\vec{E}| = \dfrac{e(R)}{4\pi\varepsilon_0 R^2} \leftrightarrow \dfrac{e}{4\pi\varepsilon(R) R^2} \right)$

Decreasing R the electric field increases faster than $1/R^2$
$\Rightarrow$ the magnitude of the charge has increased

Limits: $\begin{cases} R \to \infty \ (Q \to 0) \Rightarrow e \to -1.602\,176\,462(63) \cdot 10^{-19} C \\ R \to 0 \ (Q \to \infty) \Rightarrow e \to -\infty \end{cases}$ ("bare charge", cf. the Landau pole)

At what distance does this start to happen?

to pair produce an electron-positron pair the momentum of the "parent" electron has to change with the order of $Q = |\Delta p| \sim m_e c$

uncertainty principle gives $R \sim \Delta x \sim \dfrac{\hbar}{m_e c} \simeq 4 \cdot 10^{-13} m$

$\Rightarrow$ Quantum effects become visible for $Q \gtrsim m_e c \Leftrightarrow R \lesssim 10^{-13} m$

For all distances larger than subatomic

$$\alpha_{\mathrm{QED}}(Q=0) = \frac{e^2(Q=0)}{4\pi\varepsilon_0 \hbar c} = \frac{1}{137.035\ \ldots}$$

is correct but not for shorter distances

What now? Is the electron charge really infinite or is QED wrong?

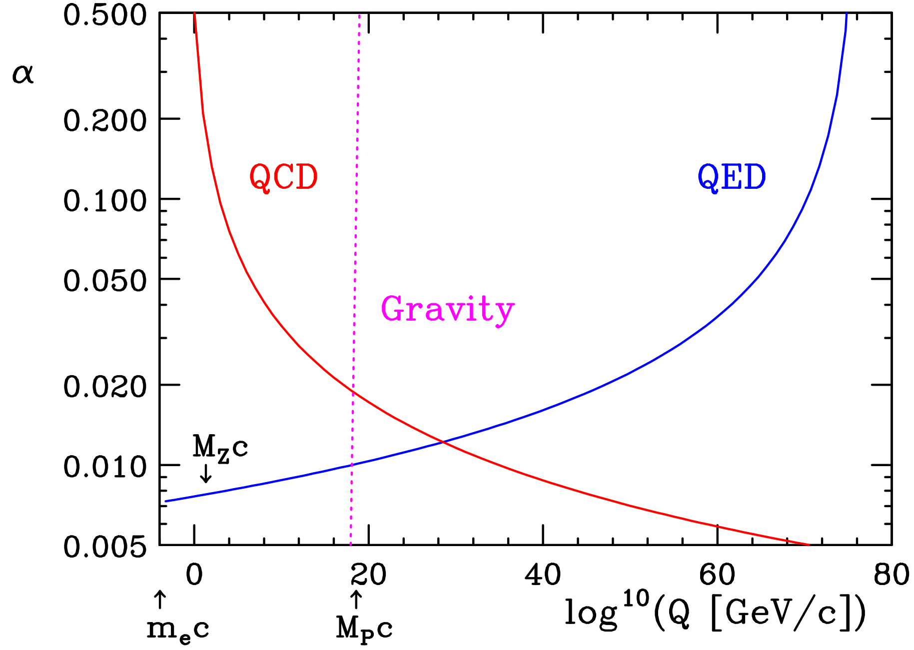
At the Planck scale ($M_P \sim 10^{19}$ GeV/c^2) gravitional interactions cannot be ignored

Gravity couples to mass or more generally speaking to energy $E = mc^2 \simeq Qc$

Newtons law:

$$F = G_N \frac{m_1 m_2}{R^2} \rightarrow G_N \frac{(Q/c)^2}{R^2} = \frac{\alpha_{\text{gravity}}}{R^2} \hbar c$$

$$\Rightarrow \alpha_{\text{gravity}}(Q) = G_N \frac{(Q/c)^2}{\hbar c}$$



Need to merge quantum mechanics and general relativity ⇒ string theory?

QED is correct for scales below the Planck scale

Quantum chromodynamics (QCD)

Describes the strong interactions of quarks and gluons

As far as we know today quarks (together with electrons) are fundamental building blocks of nature.

For exampel protons and neutrons have the properties corresponding to three-quark states: $p \sim uud$, $n \sim udd$. (Today we know of six types of quarks or flavours)

QCD and QED are very similar (SU(3) instead of U(1) gauge symmetry):

quarks $\sim$ electrons but three strong charges (colours) r, b, g instead of one ($r + g + b =$ neutral)

gluons $\sim$ photons but charged: $r\bar{g}$, $r\bar{b}$, $g\bar{b}$, $g\bar{r}$, $b\bar{r}$, $b\bar{g}$, $\ldots$: $3^2 - 1 = 8$ combinations

The potential between two colour charges in QCD ($\hbar = c = 1$):

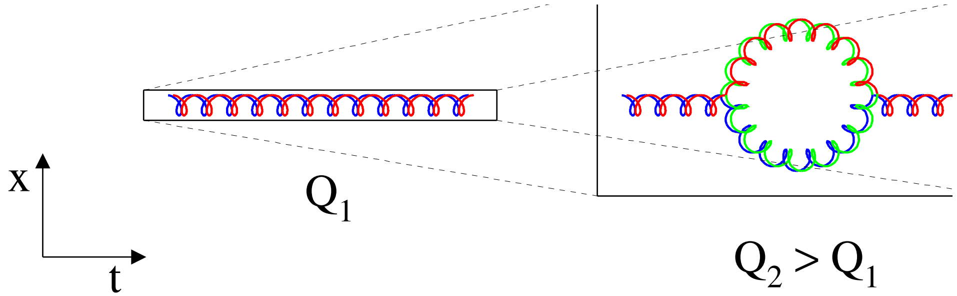
$$V(R) = -\frac{C_p \alpha_s(R)}{R} \equiv -\frac{C_p g_s^2(R)}{4\pi R}$$

$C_q = 4/3$ and $C_g = 3 \leftrightarrow$ different magnitude of quark and gluon colour charges

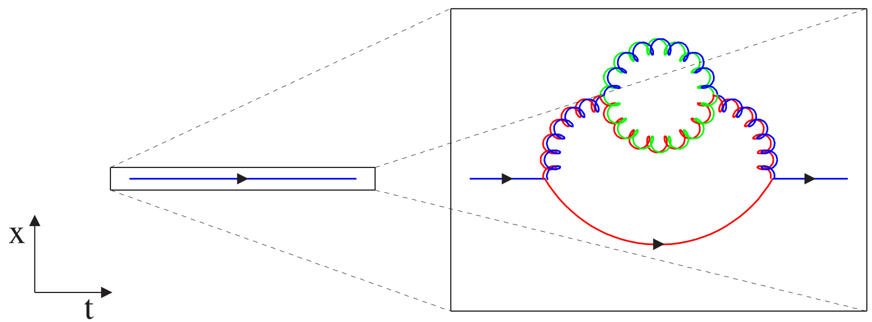
Vacuum polarisation in QCD

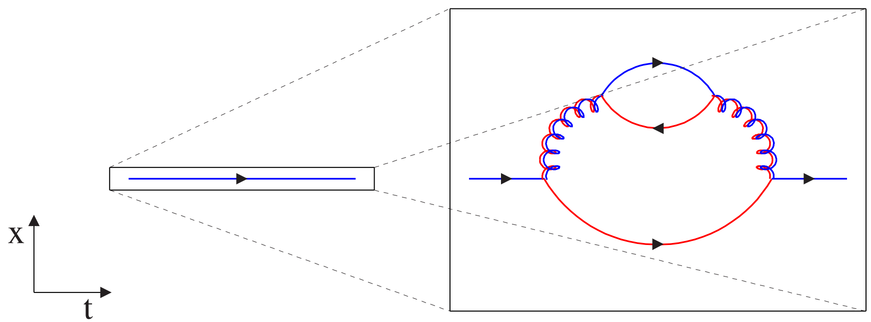
Vacuum fluctuations similar to QED but in addition:

gluon fluctuation to gluons:



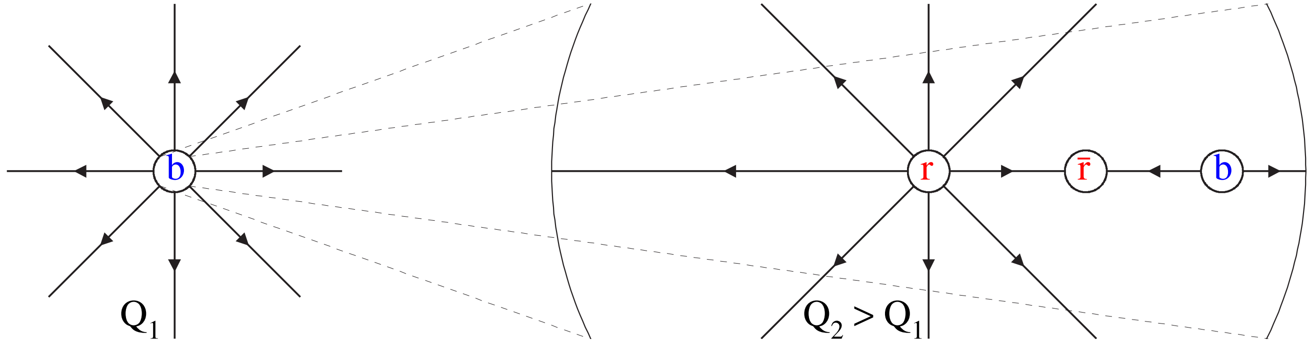
Two types of vacuum polarisation around a quark from strong interactions

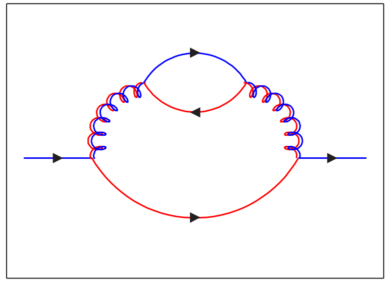




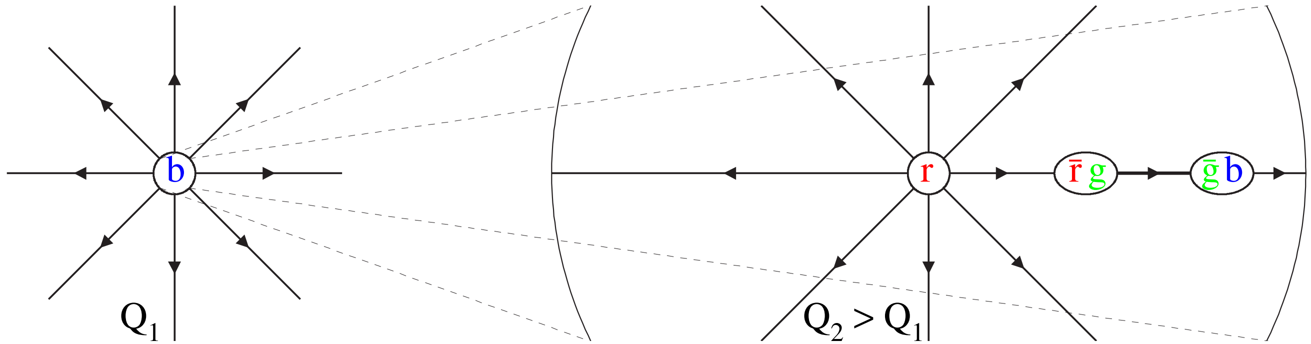
Effects of vacuum polarisation on the colour-electric field from a quark colour charge:

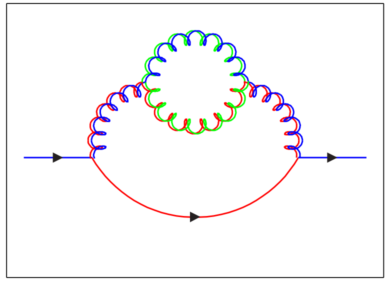
1. *screening* by quark vacuum fluctuations (as in QED)





2. *anti-screening* by gluon vacuum fluctuations (not in QED)





Net result is *anti-screening* because effect of gluons is stronger

(strictly speaking it depends on $33 - 2N_F$, where N_F=number of quark flavours)

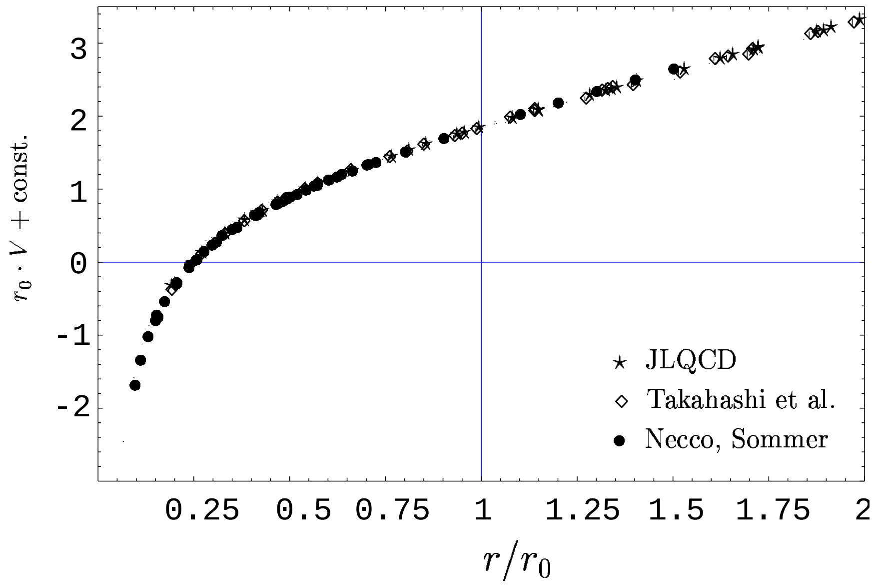
Limits: $\begin{cases} R \to 0 \ (Q \to \infty) \Rightarrow g_s \to 0 & \text{(asymptotic freedom)} \\ R \to \infty \ (Q \to 0) \Rightarrow g_s \to \infty & \text{(infrared slavery)} \end{cases}$

Infrared slavery and confinement in QCD

$g_s \to \infty \Rightarrow$ situation just as bad as in QED? No longer Coulomb potential!

For large R ($R \gtrsim 1$ fm $\Leftrightarrow Q \lesssim 0.2$ GeV) the potential becomes linear ($r_0 \sim 0.5$ fm):

$$V(R) = \kappa R\,, \qquad \kappa \simeq 1\text{GeV/fm}$$



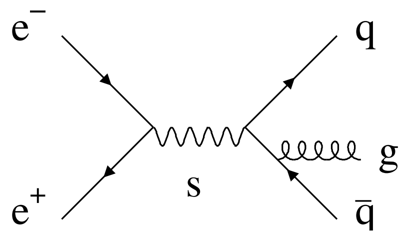
- infinite energy required to separate two colour charges
- "Explains" confinement, the non-observation of free quarks and gluons.
- Instead, only colour-neutral combinations of quarks and gluons are observed
- ex. baryons qqq, mesons $q\bar{q}$ and possibly glueballs gg, pentaquarks $qqqq\bar{q}$, . . .

Asymptotic freedom

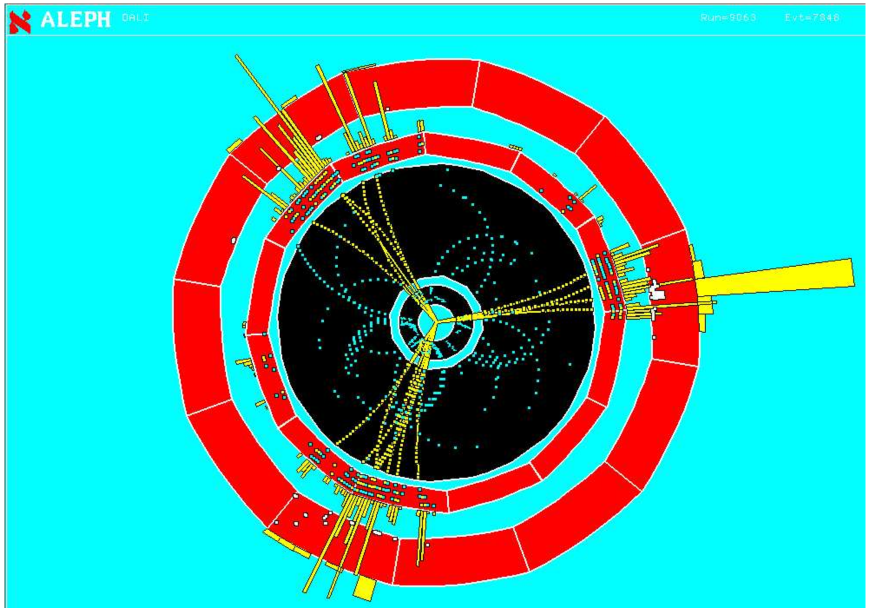
The coupling α_s is small for large momentum-transfers ($Q^2 \gtrsim 1$ GeV^2)
$\Rightarrow$ perturbative expansion in α_s possible, $\sigma = \sum_n \sigma_n \alpha_s^n$

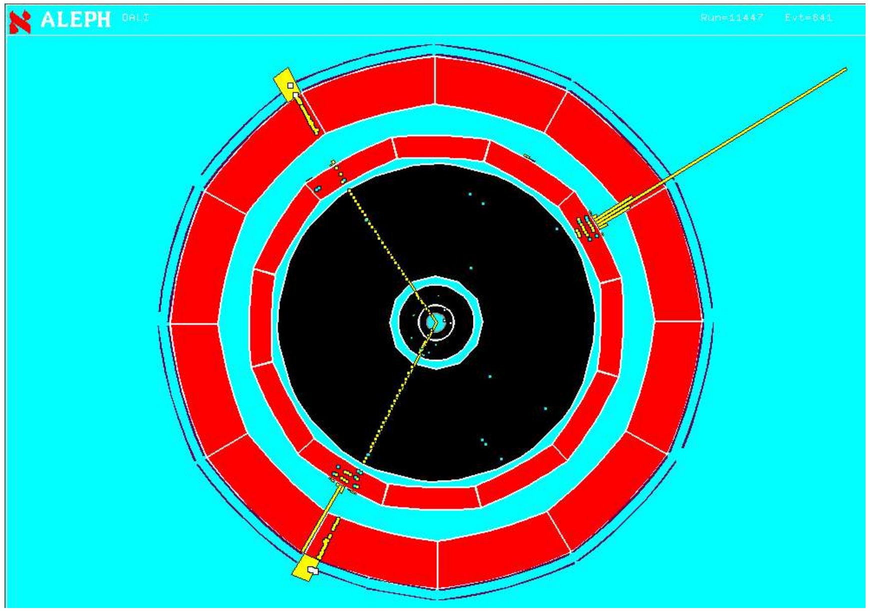
Can make calculations with quarks and gluons as if they where free particles

Ex. Three-jet events: $e^+e^- \to q\bar{q}g$
similar structure as $e^+e^- \to \mu^+\mu^-\gamma$
(discovery of gluon at DESY in 1979)

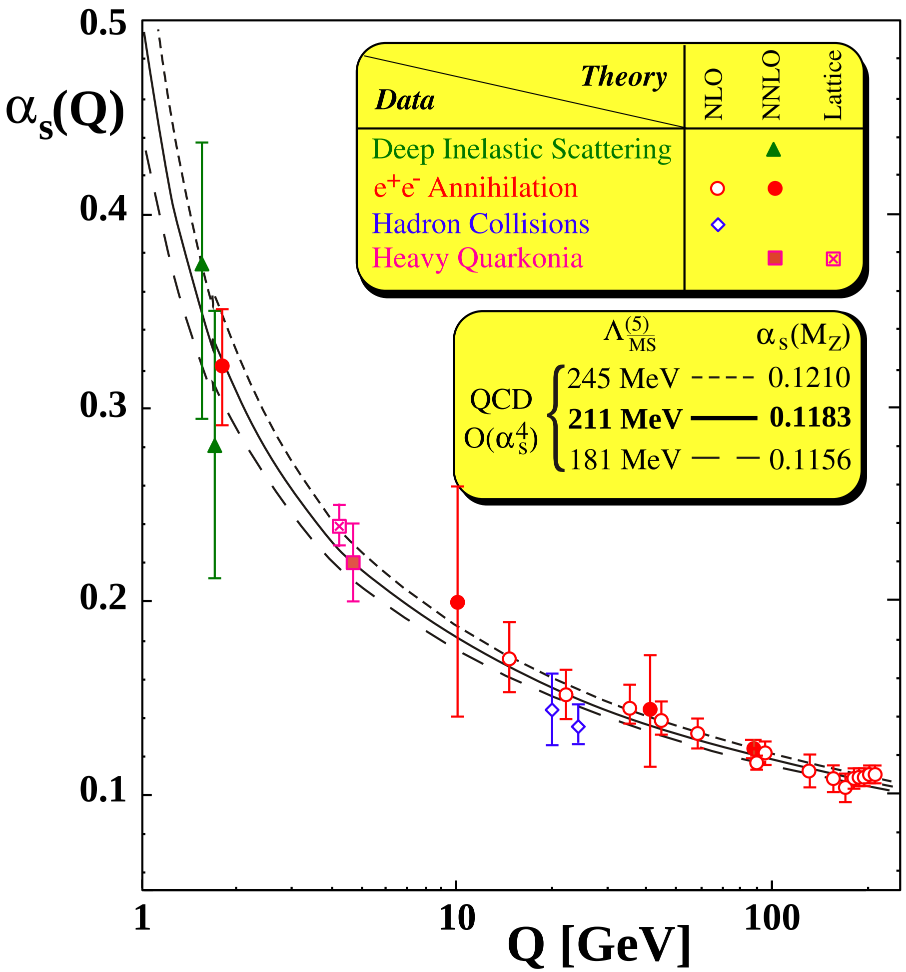


$$\alpha_s \simeq \frac{\sigma_{3-\text{jets}}}{\sigma_{\text{had}}}$$





Experimental data and theory



Evolution equation for coupling (leading order, QED and QCD)

$$\frac{d\alpha(Q^2)}{d\ln Q^2} = \beta\alpha(Q^2)^2$$

sign of $\beta \Rightarrow$ increase or decrease

$(\beta_{\mathrm{QCD}} = \frac{2N_F-33}{12\pi},\ \beta_{\mathrm{QED}} = \frac{N_{\mathrm{gen}}}{3\pi})$

Solution

$$\alpha(Q^2) = \frac{1}{-\beta\ln(Q^2/\Lambda^2)}$$

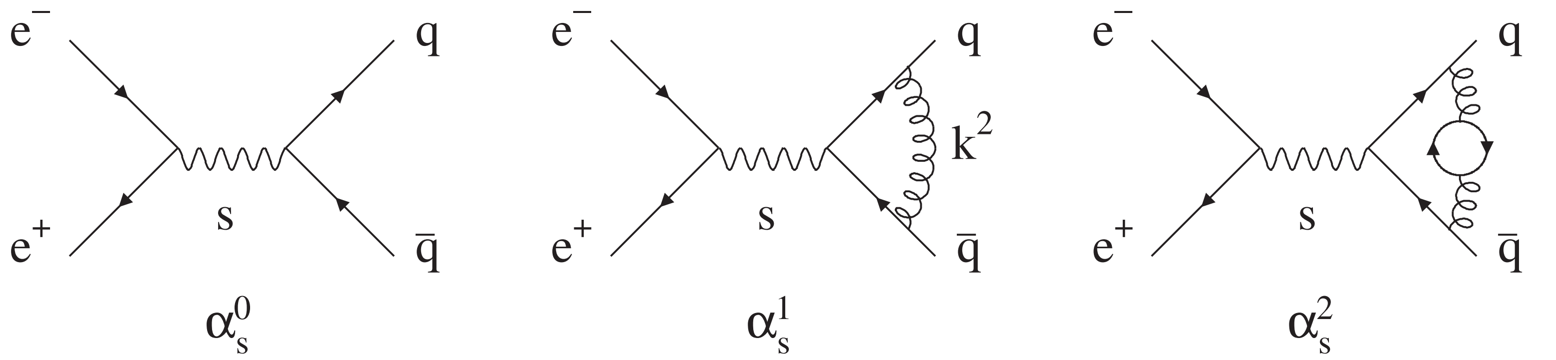
Λ small in QCD (large in QED)

Shows running of coupling in agreement with theory, $\mathcal{O}(\alpha_s^4)$!

How to do calculations with a running coupling

Ex. $e^+e^- \to$ hadrons: cross-section depends only on total energy, $\sigma(s)$

Representative Feynman diagrams in perturbative expansion



The cross-section with a running coupling:

$$\sigma(s) = \sigma_0(s)\left[1 + \int \frac{dk^2}{k^2}\mathcal{F}_1\left(\frac{k^2}{s}\right)\alpha_s(k^2) + \ldots\right]$$

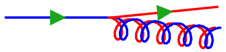
How to calculate the integral?

I. Expand in fixed coupling at some scale $\alpha_s(xs)$ (standard approach)

II. Try to perform the integral (take running seriously)

Integrating over the running coupling

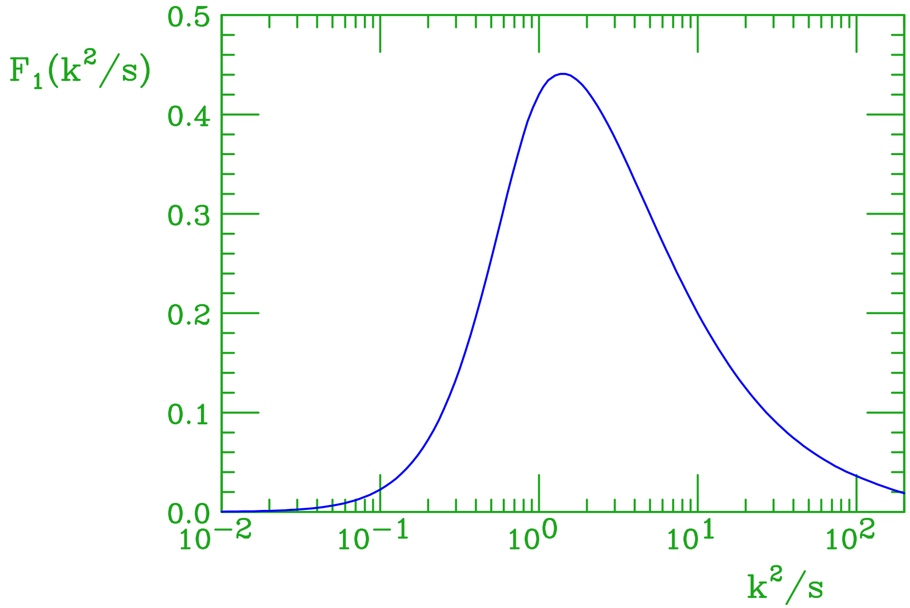
Know α_s becomes large at small scales k^2, what about $\mathcal{F}_1(k^2/s)$?

Radiation of gluons with momentum transfer smaller than the resolution cannot be observed  $\Rightarrow \mathcal{F}_1\left(\frac{k^2}{s}\right) \sim \left(\frac{k^2}{s}\right)^m$ with $m \geq 1/2$ at small k^2

Ex. $\sigma(e^+e^- \to \text{hadrons})$:

$$\mathcal{F}_1\left(\frac{k^2}{s}\right) \sim \left(\frac{k^2}{s}\right)^2$$

at small (and large) k^2

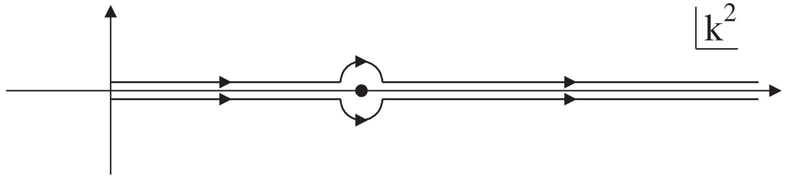


Should be possible to calculate the integral?

But, $\alpha_s(k^2) = \dfrac{1}{-\beta \ln(k^2/\Lambda^2)}$ has pole at $k^2 = \Lambda^2$! $\Rightarrow$ need to regulate the integral

$$I = \int_0^\infty \frac{dk^2}{k^2} \mathcal{F}_1\left(\frac{k^2}{s}\right) \alpha_s(k^2) = P.V. \int_0^\infty \frac{dk^2}{k^2} \mathcal{F}_1\left(\frac{k^2}{s}\right) \alpha_s(k^2) + \Delta I$$

for example by taking the principal value:



$\Rightarrow$ integral can be performed but result is ambigous, ΔI:

$$\Delta I \propto \text{residue}\left\{\int_0^\infty \frac{dk^2}{k^2} \mathcal{F}_1\left(\frac{k^2}{s}\right) \frac{1}{\ln k^2/\Lambda^2}\right\} = \mathcal{F}_1\left(\frac{\Lambda^2}{s}\right) \sim \left(\frac{\Lambda^2}{s}\right)^m = \exp\left(\frac{m}{\beta \alpha_s(s)}\right)$$

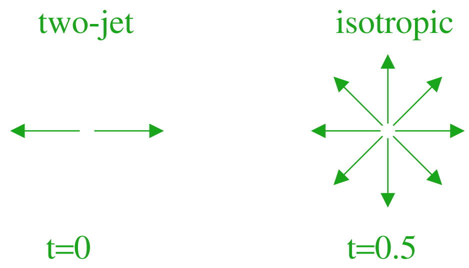
- Lacks perturbative expansion $\Rightarrow$ non-perturbative power-corrections
- power m predicted from perturbation theory

Taking the running seriously we learn something about non-perturbative effects!

How to measure α_s

Need observable which is sensitive to α_s. Ex. Thrust distribution in $e^+e^- \to$ hadrons.

$$t = 1 - \max_{\vec{n}_T} \frac{\sum_i \vec{n}_T \cdot \vec{p}_i}{\sum_i |\vec{p}_i|}$$



Fixed order perturbation theory:

(what scale to use, higher orders $\Leftrightarrow$ smaller scales, what non-perturbative corrections)

$$\frac{1}{\sigma}\frac{d\sigma}{dt}(s,t) \quad = \quad c_1(t)\alpha_s + c_2(t)\alpha_s^2$$

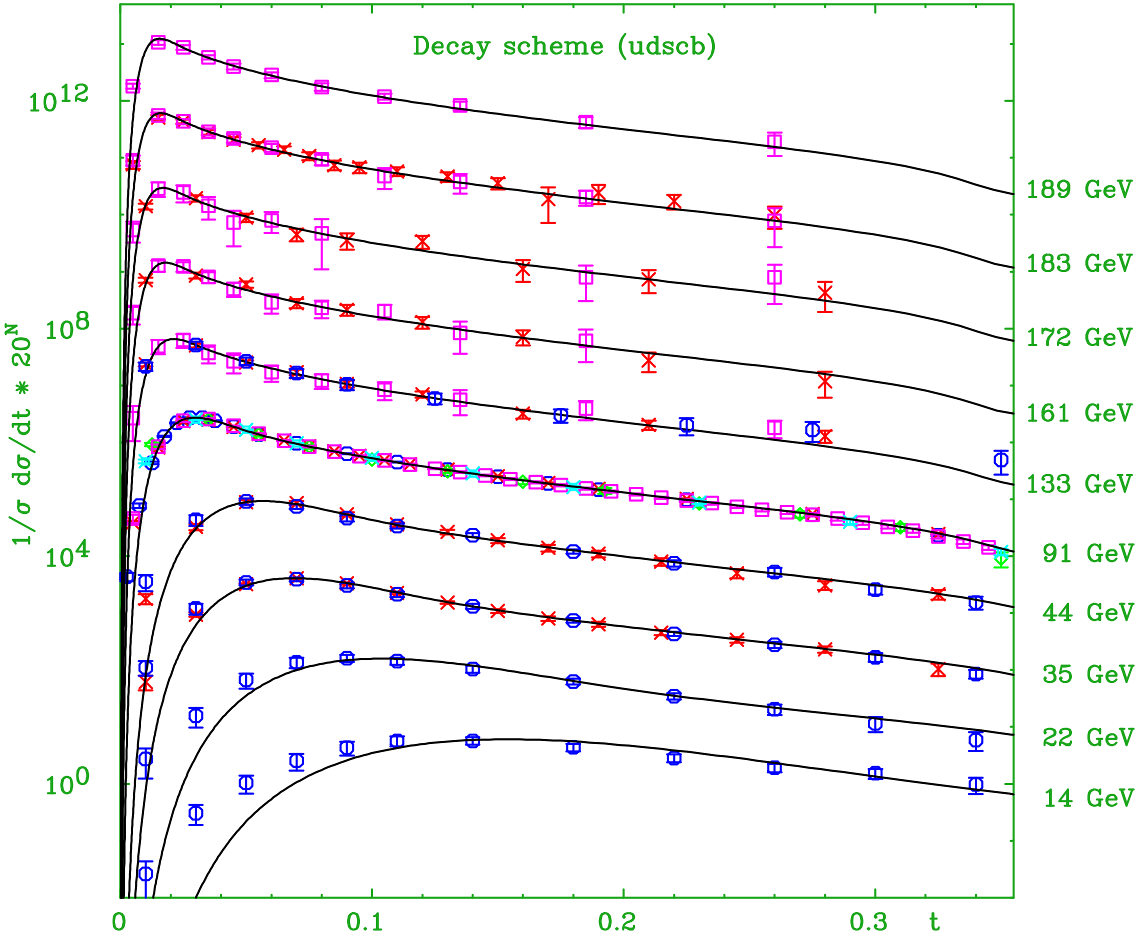
As an integral over the running coupling:

$$\frac{1}{\sigma}\frac{d\sigma}{dt}(s,t) \quad = \quad \int_C \frac{d\nu}{2\pi i\nu} e^{\nu t} \exp \underbrace{\left[\int d\tau \left(e^{-\nu\tau} - 1\right) \int_{\tau^2 s}^{\tau s} \frac{dk^2}{k^2} \mathcal{F}_1\left(\frac{k^2}{s}\right) \alpha_s(k^2) \right]}_{\text{gives power and t-dependence of non-pert. corr.}}$$

non-perturbative effects taken into account using essentially just two parameters

(normally one uses Monte-Carlo programs with tens of relevant parameters)

$\Rightarrow$ possible to fit hundreds of data points for the thrust-distribution at 10 different energies

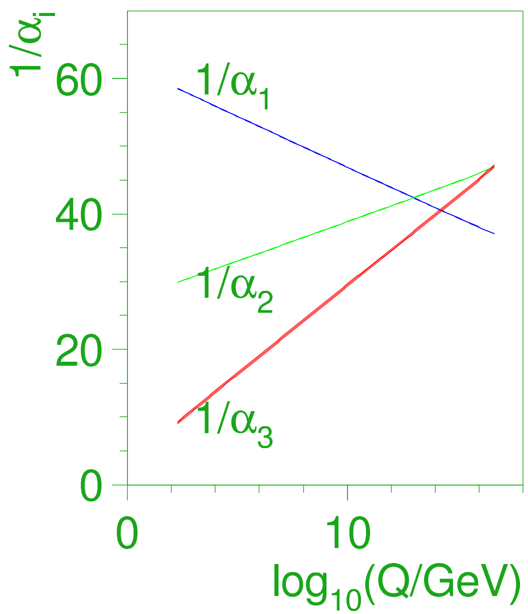


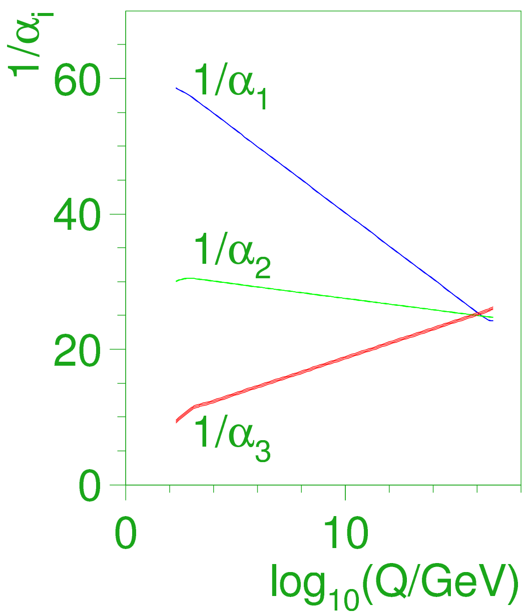
$$\Rightarrow \alpha_s(M_Z) = 0.110 \pm 0.005$$

Grand unification

Can we use the running of couplings to say something about what happens at large scales? Are all interactions unified into one?

- Since α_s decreases with Q and $\alpha_{\rm QED}$ increases they will meet at some scale!
- The weak coupling also depends on Q – will it meet the others?





Normalisation of couplings from unified group, e.g. SU(5) or SO(10):

$$\alpha_1 = \frac{5\alpha_{\rm QED}}{3\cos^2\theta_W}$$

$$\alpha_2 = \frac{\alpha_{\rm QED}}{\sin^2\theta_W}$$

$$\alpha_3 = \alpha_s$$

(left) the couplings (almost) meet before the Planck scale

(right) including supersymmetric particles the agreement is even better

Conclusions

Quantum fluctuations polarise the vaccum and makes the interaction strength (coupling) depend on the resolution (energy)

α_{QED} increases with resolution, α_s decreases

running of couplings established experimentally

α_s small – meaningful to do calculations with quarks and gluons

can write observables as integrals over running coupling

extrapolating to higher energies (the GUT-scale) different couplings become of similar size $\Rightarrow$ underlying unified theory?

at even higher energies (the Planck scale) gravitational effects have to be taken into account (string theory?)