Problems Chapter 6, QFT 2010

1. Show the Gordon identity

$$\bar{u}(p')\gamma^\mu u(p) = \bar{u}(p')\left[\frac{p+p'}{2m} + \frac{i\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

2. Show that

$$\begin{aligned}
\frac{1}{AB} &= \int dx dy \delta(x+y-1)\frac{1}{(xA+yB)^2} \\
\frac{1}{ABC} &= \int dx dy dz \delta(x+y+z-1)\frac{2}{(xA+yB+zC)^3}
\end{aligned}$$

3. Show that

$$\begin{aligned}
&\bar{u}(p')\left\{[-y\not{q}+z\not{p}]\gamma^\mu[(1-y)\not{q}+z\not{p}] + m^2\gamma^\mu - 2m[(1-2y)q^\mu + 2zp^\mu]\right\}u(p) = \\
&= \bar{u}(p')\left\{\gamma^\mu[(1-x)(1-y)q^2 + (1-2z-z^2)m^2] + (p'+p)^\mu m z(z-1) + \right. \\
&\qquad \left. + q^\mu m(z-2)(x-y)\right\}u(p)
\end{aligned}$$

   and that the coefficient of the q^μ term vanishes after integration over x and y.

4. Use a Wick rotation to show that

$$\begin{aligned}
\int \frac{d^4\ell}{(2\pi)^4}\frac{1}{(\ell^2-\Delta+i\varepsilon)^m} &= \frac{i(-1)^m}{(4\pi)^2}\frac{1}{(m-2)(m-1)}\frac{1}{\Delta^{m-2}} \\
\int \frac{d^4\ell}{(2\pi)^4}\frac{\ell^2}{(\ell^2-\Delta+i\varepsilon)^m} &= \frac{i(-1)^{m+1}}{(4\pi)^2}\frac{1}{(m-3)(m-2)(m-1)}\frac{1}{\Delta^{m-3}}
\end{aligned}$$

5. Use the method of Feynman parameters to show that $I(v,v') = 2f_{\rm IR}(q^2)$ where

$$\begin{aligned}
I(v,v') &= \int \frac{d\Omega_{\hat{k}}}{4\pi}\left(\frac{2\,p\cdot p'}{p\cdot k\, p'\cdot k} - \frac{m^2}{(p\cdot k)^2} - \frac{m^2}{(p'\cdot k)^2}\right) \\
2f_{\rm IR}(q^2) &= \int_0^1 \frac{2m^2-q^2}{m^2-\xi(1-\xi)q^2}d\xi - 2 = \int_0^1 \frac{2\,p\cdot p'}{(\xi^2+(1-\xi)^2)m^2 + 2\xi(1-\xi)\,p\cdot p'}d\xi - 2
\end{aligned}$$

   and that for $-q^2 \gg m^2$

$$f_{\rm IR}(q^2) = \log\frac{-q^2}{m^2}$$

Please send an email to Johan.Rathsman@physics.uu.se before the problem solving session stating one problem that you would like to solve on the blackboard and one problem that you would like to see the solution to. (Click an the email address to get a preformated mail.)