Hand-in Exercise nr 5 Quantum Field Theory 10

Non-QED contributions to (g − 2)

The axion is pseudo-scalar particle that is predicted by the Peccei-Quinn mechanism for solving the so called strong CP-problem, *i.e.* why the combined CP-symmetry is preserved by the strong interaction. In a certain class of axion models (called DFSZ) the axion particle ϕ_a couples to charged leptons ℓ, *i.e.* an electron or a muon, with mass m_ℓ, according to

$$H_{\rm int} = \int d^3x \frac{i\lambda_\ell}{\sqrt{2}}\,\phi_a\,\bar{\psi}\,\gamma^5\,\psi$$

where $\lambda_\ell = m_\ell \dfrac{m_a \cos^2\beta}{m_\pi f_\pi}$ with m_a being the axion mass, $\cos\beta$ a free parameter in the range $0.01 \lesssim \cos\beta \lesssim 1$, and $m_\pi = 135$ MeV and $m_\pi = 92$ MeV the pion mass and decay constants respectively.

a) Compute the contribution of a virtual axion to the anomalous magnetic moment $a = (g-2)/2$ of a charged lepton in the limit $m_\ell \gg m_a$. Also verify the Ward-identity, *i.e.* that the coefficient of the q^μ-term vanishes.

   Hints: A psedo-scalar particle has the same propagator as a scalar one. When calculating the numerator start with getting rid of the γ^5-matrices by commuting one of hem until it stands next to the other.

b) The current experimental uncertainty on the muon anomalous magnetic moment a_μ is $\approx 1\cdot 10^{-9}$. What limit does this give on the axion mass if $\cos\beta = 1$?

To isolate the contribution to F_2 from the numerator you may find it useful to use the following projection operator,

$$F_2(q^2) = \frac{m^2}{q^2(4m^2-q^2)}\mathrm{Tr}\left[\left(\gamma_\mu - \frac{q^2+2m^2}{4m^2-q^2}\,\frac{p'_\mu+p_\mu}{m}\right)(\not{p}'+m)\Gamma^\mu(p,p')(\not{p}+m)\right]$$

where $p^2 = (p')^2 = m^2$, $q = p' - p$ and $\Gamma^\mu(p,p') = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)$. If you use this projection operator you should first prove it by calculating $\mathrm{Tr}\left[\gamma_\mu(\not{p}'+m)\Gamma^\mu(p,p')(\not{p}+m)\right]$ and $\mathrm{Tr}\left[(p'_\mu+p_\mu)(\not{p}'+m)\Gamma^\mu(p,p')(\not{p}+m)\right]$.

A very efficient tool for calculating traces is the program FORM which you can find at `http://www.nikhef.nl/~form/`

You can find a FORM program that calculates the contribution to F_2 in QED on the course homepage.

Note: To be handed in on the latest on June 7 (but prefereably June 2) to be given back on the latest June 11.