2006-11-20

Hand-in exercises in the course QCD@Colliders, HT 2006

2. Asymptotic freedom and confinement

A. The running coupling:

Consider the expansion of the dimensionless observable
$R(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)) = R_1\alpha_s(\mu^2) + R_2(\frac{Q^2}{\mu^2})\alpha_s{}^2(\mu^2) + \ldots$

Show that R_1 is independent of μ and that $R_2(\frac{Q^2}{\mu^2}) = R_2(Q^2 = \mu^2) - R_1 b \ln(\frac{Q^2}{\mu^2})$

B. The β-function:

Show that the first two coefficients in the β-function, β_0 and β_1, are the same in all renormalisation schemes

C. The Λ parameter and quark masses:

Show that $\Lambda^2_{\overline{\rm MS}} = \Lambda^2_{\rm MS} \exp\{\ln(4\pi) - \gamma_E\}$

Use the leading order expression for α_s in terms of the Λ parameter to **derive** the relations between $\Lambda(5)$, $\Lambda(4)$ and $\Lambda(3)$ that follow from demanding that the coupling is continous at the quark mass-thresholds. Starting from $\Lambda(5)_{\overline{\rm MS}} = 200$ MeV and using $m_b = 4.5$ GeV and $m_c = 1.5$ GeV **calculate** $\Lambda(3)_{\overline{\rm MS}}$. Finally use this result to **calculate** $\Lambda(3)_{\rm MS}$. **Compare** your results with the notion that "the QCD scale Λ" is given by the inverse radius if a typical hadron, $r_p^{-1} \sim 200$ MeV.

D*. Dimensional regularization:

Show that the integral $\int \frac{d^D k}{k^2} = 0$ in dimensional regularisation. (Hints: perform a Wick rotation in the complex k_0-plane, remember that $d^D k = i d^D K = i K^{D-1} dK d\Omega_D$, use $\int d\Omega_D = \frac{2\pi^{D/2}}{\Gamma(D/2)}$, divide the integral over K into an infrared and an ultraviolet region with different D and finally analytically continue the two results to a common D.)

E**. Asymptotic freedom:

Calculate the non-abelian contribution to β_0 from the static quark potential, $V(Q^2) = -C_F 4\pi\alpha_V(Q^2)/Q^2$, using Feynman gauge. The Feynman rules are given by the $m \to \infty$ limit of the heavy quark rules and the diagrams which contribute in Feynman gauge are illustrated below. Note that in dimensional regularisation diagram (c) does not contribute (why?), that it is only the non-abelian part of (d) and (e) that contributes, and that in general covariant gauge there is also a diagram with the three-gluon vertex that contributes.