Introduction to MC@N LO

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QCD@Colliders
Student Presentation
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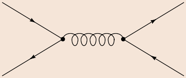
Process generation in standard MC

1. The hard event is generated from the LO x-section

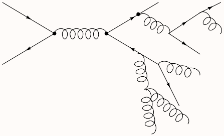
Born-level Matrix Element calculations are "simple":



2. More quarks and gluons produced with parton shower

Leading log Sudakov resummation of collinear singularities

$\Delta(t) = \exp\left\{ -\int_{t_0}^{t} \frac{dt'}{t'} \int dz \frac{\alpha_s}{2\pi} \hat{P}(z) \right\}$



In addition, hadronization and resonance decays are added to produce full (hadronic) events.

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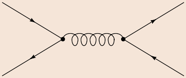

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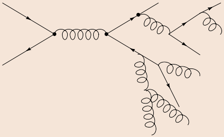

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Process generation in standard MC

Limitations with this approach

- Total rates only accurate to leading order (need K-factors, which cannot be obtained event by event)
- Standard MC's don't do multi-jet observables very well. This may have severe implications on future search strategies
- No meaningful measure of the QCD scale dependences can be obtained

Solutions

Want to use both higher order matrix elements and parton showers simultaneously, need smooth matching and no double-counting.

- Matrix element corrections (old approach)
- Matching (→ See David's talk on thursday)
- MC@NLO

Definition of an MC@NLO

What to expect from MC@NLO

A possible minimal definition of an MC@NLO would be:

- *A parton shower Monte Carlo where the hard process is calculated including full QCD corrections (real and virtual) up to Next-to-Leading Order in perturbation theory.*
- As such, it should output complete *events* like any normal MC
- In addition, it should preferrably be linked to, or contain, all the useful features of ordinary MC programs

MC@NLO implementations

Original MC@NLO

MC@NLO approach by Frixione and Webber (2002)
Used in MC@NLO package:
`http://www.hep.phy.cam.ac.uk/theory/webber/MCatNLO/`

Upcoming competitor

Alternative POWHEG approach by Nason (2006)
No public code available yet

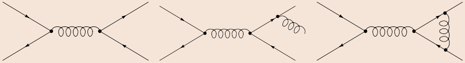
References

S. Frixione, B. Webber [hep-ph/0204244]
S. Frixione, P. Nason and B. Webber [hep-ph/0305252]
S. Frixione, B. Webber [hep-ph/0612272]
P. Nason, G. Ridolfi [hep-ph/0606275]

Process generation in MC@NLO

1. Complete NLO x-section is used

Calculations become more involved



Divergences cancel between real and virtual contributions

2. Add and subtract MC counterterms

Avoids double-counting. Analytic expression for the first shower emission is needed.

3. Hard configuration showered as usual

Again resummation of collinear singularities, *but*
hardest emission now described by NLO matrix element

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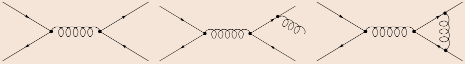

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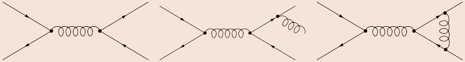

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Example implementation: MC@NLO toy model

Assume a system can radiate massless particles with energy fraction x, $0 \leq x \leq x_s \leq 1$ with x_s the system energy before radiation. The massless particles cannot radiate any further.

$$\left(\frac{d\sigma}{dx}\right)_B = B\delta(x)$$

$$\left(\frac{d\sigma}{dx}\right)_V = a\left(\frac{B}{2\epsilon} + V\right)\delta(x)$$

$$\left(\frac{d\sigma}{dx}\right)_R = a\frac{R(x)}{x}$$

In these expressions, a is the coupling and ϵ the parameter of dimensional regularization ($D = 4 - 2\epsilon$).

$$\left(\frac{d\sigma}{dx}\right)_{NLO} = \left(\frac{d\sigma}{dx}\right)_B + \left(\frac{d\sigma}{dx}\right)_V + \left(\frac{d\sigma}{dx}\right)_R$$

Example implementation: MC@NLO toy model

Any infrared-safe observable can formally be obtained from

$$< O >= \lim_{\epsilon\to 0} \int_0^1 dx\, x^{-2\epsilon} O(x) \left(\frac{d\sigma}{dx}\right)_{NLO}$$

which after some manipulation can be written as

$$< O >= \int_0^1 dx \left[O(x)\frac{aR(x)}{x} + O(0)\left(B + aV - \frac{aB}{x}\right)\right]$$

This is all general stuff. There is no MC@NLO or even MC yet. In a MC treatment, multiple emissions are generated according to the Sudakov form factor:

$$\Delta(x_1, x_2) = \exp\left(-a\int_{x_1}^{x_2} dz \frac{Q(z)}{z}\right)$$

where $Q(z)/z$ is the “splitting” function of this system

Example implementation: MC@NLO toy model

Introduce the notation $I_{MC}(O, x_M)$ for the distribution of an observable O, obtained with the MC started at a scale x_M. All events are given equal weight.
We can now attempt to substitute $O(x) \rightarrow I_{MC}(O, x_M(x))$ in our general NLO expression:

$$\frac{d\sigma}{dO} = \int_0^1 dx \left[I_{MC}(O, x_M(x)) \frac{aR(x)}{x} + I_{MC}(O, 1) \left(B + aV - \frac{aB}{x} \right) \right]$$

Suggesting a scheme where x is generated, as input for two separate MC events with different starting scales and different weights.
Unfortunately this idea fails, since the weights diverge in the limit $x \rightarrow 0$. Furthermore, this approach suffers from double-counting (not shown here).

Example implementation: MC@NLO toy model

The solution is to add something, and to subtract something else, namely the "NLO" x-section of the parton shower $aBQ(x)/x$:

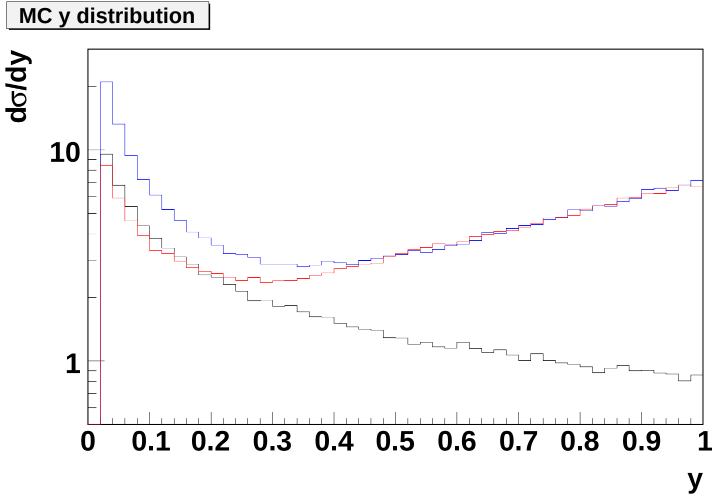
$$\left(\frac{d\sigma}{dO}\right)_{MC@NLO} = \int_0^1 dx \left[I_{MC}(O, x_M(x)) \frac{a(R(x) - BQ(x))}{x} + \right.$$
$$\left. + I_{MC}(O,1)\left(B + aV + \frac{aB(Q(x)-1)}{x}\right)\right]$$

which can be shown to give the desired result correct to $O(a)$. This is the MC@NLO master equation for the toy model. Let's now have a look at some results for the interesting variables

$$y = \max\{x_1, x_2, ..., x_n\}$$

$$z = x_i \quad \forall i$$

Distribution of y (hardest photon energy)

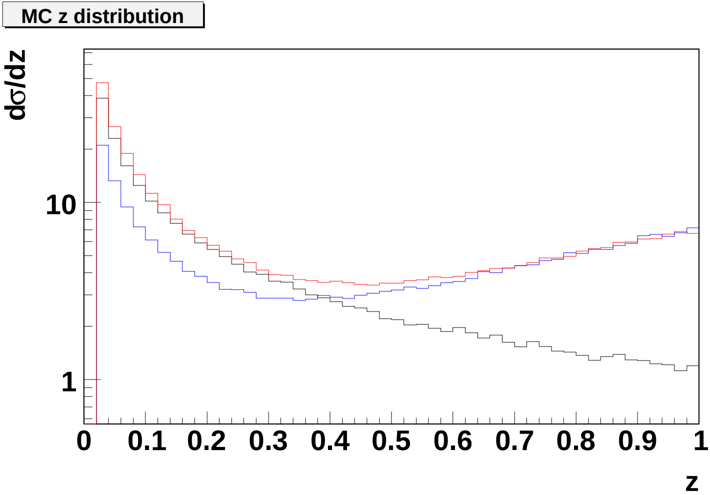


— Pure MC

— Analytic NLO

— MC@NLO

Distribution of z (all photon energies)



— Pure MC

— Analytic NLO

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Using MC@NLO

From the user's perspective...

MC@NLO is implemented as a standalone addition to HERWIG.
→ HERWIG is required, no PYTHIA :(

- First run MC@NLO to produce the "hard" and "soft" event samples
- These are then fed through a LHA interface into HERWIG for showering and hadronisation

Negative weights

Whenever quantum amplitudes are summed, interference contributions may be positive, negative or zero. As a result, events from MC@NLO might end up having negative weights.
These must be included and should be treated exactly as the normal events, keeping the sign of the weight.
POWHEG different → Watch out for updates on this!

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NLO Processes implemented so far

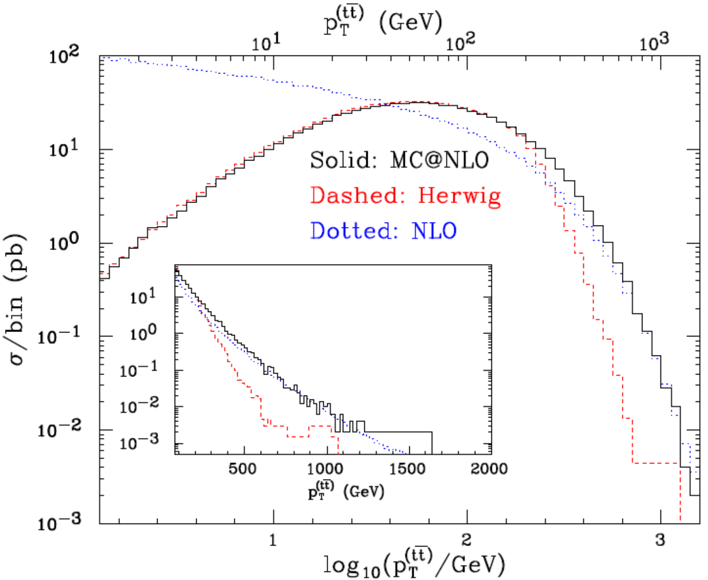
List of processes available in MC@NLO 3.3

$H_1 H_2 \rightarrow \gamma^*/Z^0, Z^0 Z^0, W^+ W^-, W^+ Z^0, W^- Z^0$
$H_1 H_2 \rightarrow t, \bar{t}, t\bar{t}, b\bar{b}$
$H_1 H_2 \rightarrow H^0, H^0 W^+, H^0 W^-, H^0 Z^0$

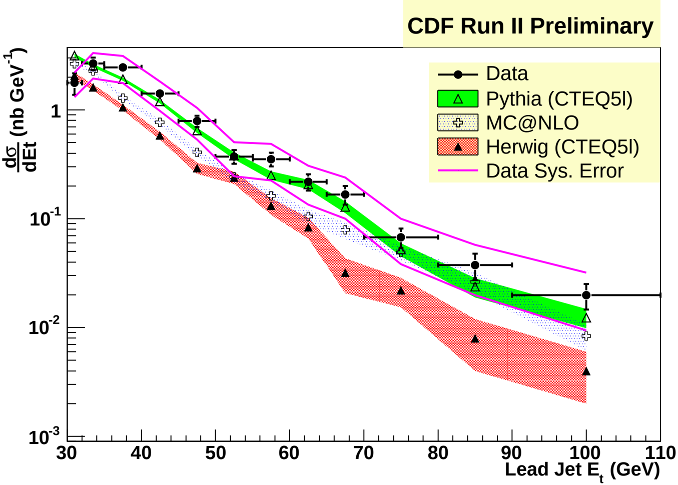
Note also...

... that spin correlations are included in many cases for vector boson- and top quark decays.

Simulated $t\bar{t}$ production



$b\bar{b}$ jet production



Summary

Final points

- Through the MC@NLO approach it is possible to combine the benefits of fixed order (NLO) calculations with those of parton shower MCs in a consistent manner
- MC@NLO code is already out there, and it is free to use!
- The number of available processes increases with time...
- ... but not so fast though since NLO calculations are harder than LO ones
- More comparisons with real data needed to favor relevance
- Alternative solutions without negative weights are coming