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# Top-Quark Decays and QCD corrections

QCD@Colliders
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# Outline

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- SM Top Decays
- QCD corrections
  - $O(\alpha_s)$
  - $O(\alpha_s^2)$
- Bonus material: “Unconventional decays”

# SM Top Decays

Predominantly: $t \to W^+ b$

- $W^+ \to l^+ \nu$ (11% per lepton)
- $W^+ \to q\,\bar{q}$ (33 % uq, 33% cq)

# SM Top Decays

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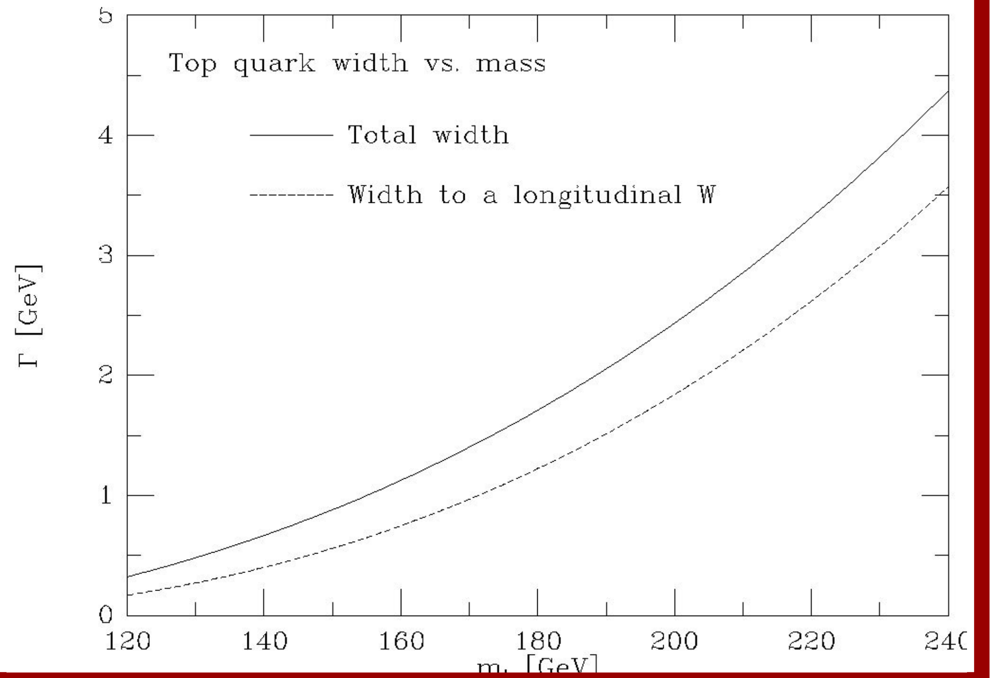
## Polarization of W: left-handed (L) or longitudinal (0)

$$\sum |M_L|^2 = \frac{2G_F m_t^4}{\sqrt{2}} |V_{tb}^2| 2x^2(1-x^2+y^2)$$

$$\sum |M_0|^2 = \frac{2G_F m_t^4}{\sqrt{2}} |V_{tb}^2| (1-x^2-y^2(2+x^2-y^2))$$

where x=M(W)/m(t), y=m(b)/m(t).

$$\frac{\Gamma(t \to bW_0)}{\Gamma(t \to bW)} = \frac{m_t^2}{m_t^2 + 2M_W^2}$$



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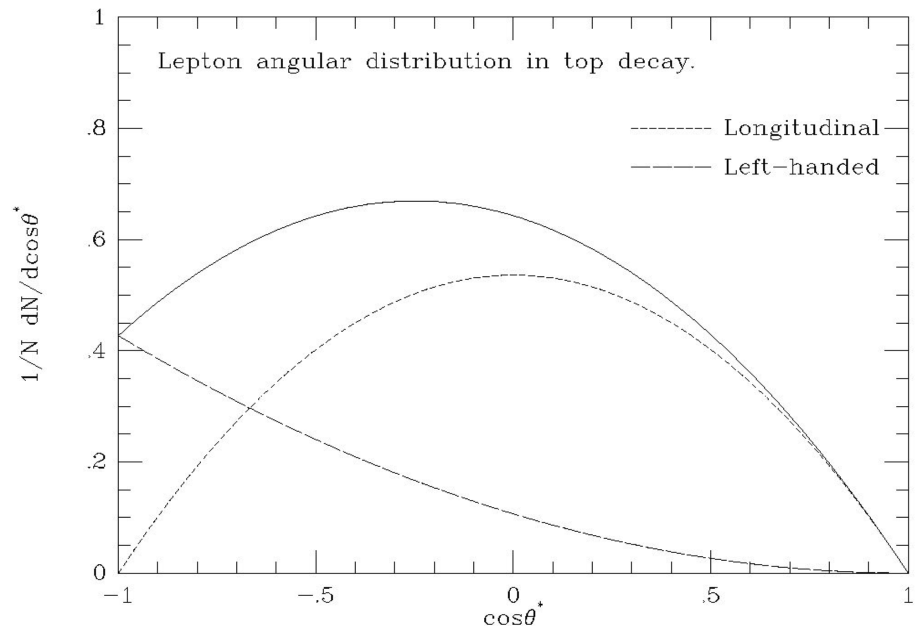
# SM Top Decays

Angular distribution of products: depends on W polarization

$$\cos(\theta_e) \approx \frac{4b \cdot e}{m_t^2 - M_W^2}$$

Angle of $l^+$ in rest frame of W w.r.t. original direction of W

Different distribution: potential sign of physics beyond the SM



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# SM Top Decays

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Asuming m(b), m(W) → 0, the total width becomes

$$\Gamma_0(t \rightarrow bW) = \frac{G_F m_t^3}{8\pi\sqrt{2}} |V_{tb}^2| \approx 1.76\,GeV \left(\frac{m_t}{175\,GeV}\right)^3$$

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Only one power of the Fermi constant: "Semi-weak" decay rate

# SM Top Decays

Large mass & semi-weak decay rate:

Total decay width bigger than typical hadronic scale

Lifetime: order of $10^{-25}$ sec

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**=> top decays before hadronization**

i.e. no “toponium” or other hadrons with top-quarks!

# SM Top Decays

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**Top quark is as close as we can get to a free quark!**

# QCD Corrections

$$\Gamma(t \rightarrow bW) = \Gamma_0 \left[ A^{(0)} + \frac{\alpha_s}{\pi} C_F A^{(1)} + \left( \frac{\alpha_s}{\pi} \right)^2 A^{(2)} + \cdots \right]$$

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$A^{(0)}$: Correction for non-zero $M_W$

$A^{(1)}$: Exact result by M. Jezabek & J.H. Kühn in 1988

$A^{(2)}$: Three different calculations

- A.Czarnecki & K.Melnikov (1998), asuming $M_W \to 0$
- K.G.Chetyrkin, R.Harlander, T.Seidensticker & M.Steinhauser (1999), Padé approx., no asumption for $M_W$
  - Using their notation in the following
- M.Slusarczyk (2004) analytically
- Results agree with each other ☺

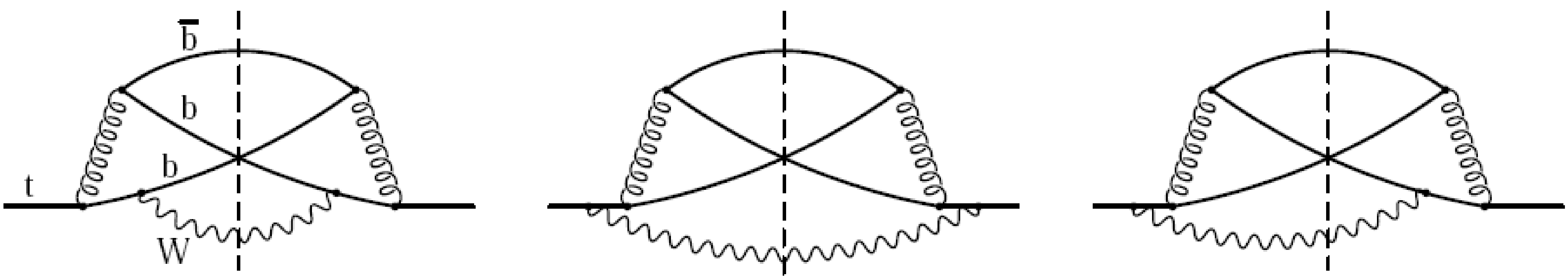
# QCD Corrections - O($\alpha_s$)

$$A^{(1)} = \frac{5}{4} - \frac{\pi^2}{3} + \frac{3}{2}\frac{M_W^2}{M_t^2} + \frac{M_W^4}{M_t^4}\left(-6 + \pi^2 - \frac{3}{2}\ln\frac{M_t^2}{M_W^2}\right)$$

- M.Jezabek and J.H.Kühn, Nucl Phys **B314**, 1 (1989)
- Calculated "using standard techniques"...
- Exact result

# QCD Corrections - O($\alpha_s^2$)

- Quite complicated situation
- No exact result available*
- Calculations invovle some non-planar diagrams...




* : According to a paper from 1999...

# QCD Corrections - $O(\alpha_s^{\,2})$

$$A^{(2)} = C_F^2 A_A^{(2)} + C_A C_F A_{NA}^{(2)} + C_F T n_l A_l^{(2)} + C_F T A_F^{(2)}$$

## Methods used

*Czarnecki & Melnikov*:

- Non-planar diagrams: Expand around $m_b = m_t/3$ in variable $\delta = 1 - 3(m_b/m_t)$
- W massless

# QCD Corrections - O($\alpha_s^2$)

$$A^{(2)} = C_F^2 A_A^{(2)} + C_A C_F A_{NA}^{(2)} + C_F T n_l A_l^{(2)} + C_F T A_F^{(2)}$$

## Methods used

*Chetyrkin et al*:

- Expand each term in $(M_w/M_t)^2$

$$A_i^{(2)} = A_i^{(2)}|_{M_W=0} + \frac{M_W^2}{M_t^2} A_i^{(2)}|_{M_W^2} + \frac{M_W^4}{M_t^4} A_i^{(2)}|_{M_W^4} + \cdots$$

- Calculate the values using Padé approximants

# QCD Corrections - O($\alpha_s^2$)

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$$A^{(2)} = C_F^2 A_A^{(2)} + C_A C_F A_{NA}^{(2)} + C_F T n_l A_l^{(2)} + C_F T A_F^{(2)}$$

## Methods used

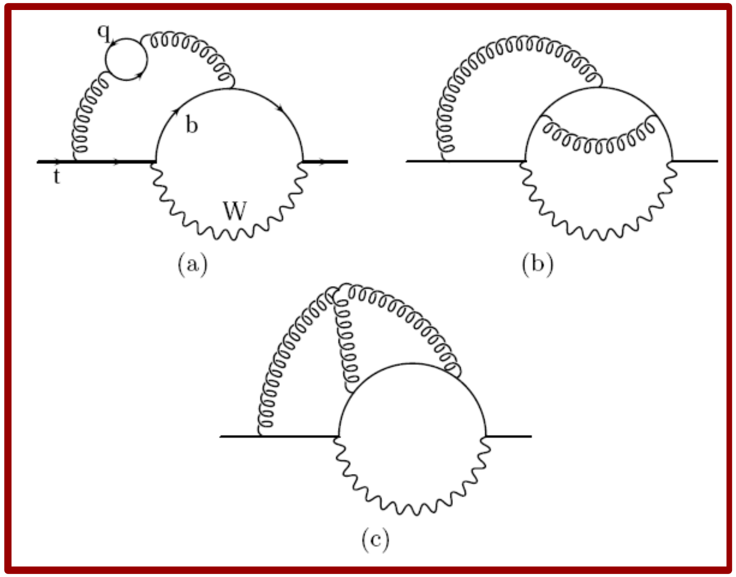
*Slusarczyk*:

- Use same expansion
- Calculate coefficients analytically
- Use the optical theorem
  - Need to compute imaginary parts of three-loop self-energy diagrams
  - Various cuts of these diagrams correspond to 2-loop virtual corrections / emission of 1 or 2 real quanta

# QCD Corrections - $O(\alpha_s^2)$

$$A^{(2)} = C_F^2 A_A^{(2)} + C_A C_F A_{NA}^{(2)} + C_F T n_l A_l^{(2)} + C_F T A_F^{(2)}$$

## Methods used

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# QCD Corrections - $O(\alpha_s^2)$



Results of all three methods are quite close

$A^{(2)}=$

- -16.7(5)
- -15.6(1.1)
- -15.5(1)

(for $M_t$=175 GeV, $M_W$=80.4 GeV, $\alpha_s(M_t^2)$=0.11)

# QCD Corrections: Numerically

$$\Gamma(t \to bW) = \Gamma_0 \left[ A^{(0)} + \frac{\alpha_s}{\pi} C_F A^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 A^{(2)} + \cdots \right]$$

$A^{(0)}$: Correction for non-zero $M_W$ 0.8852

$A^{(1)}$: M. Jezabek & J.H. Kühn -2.22 ~-10%

$A^{(2)}$: Three different calculations

- A.Czarnecki & K.Melnikov -16.7(5)
- K.G.Chetyrkin et al -15.6(1.1) ~-2%
- M.Slusarczyk -15.5(1)

In total: $\Gamma(t \to bW) = \sim 0.78\Gamma_0$

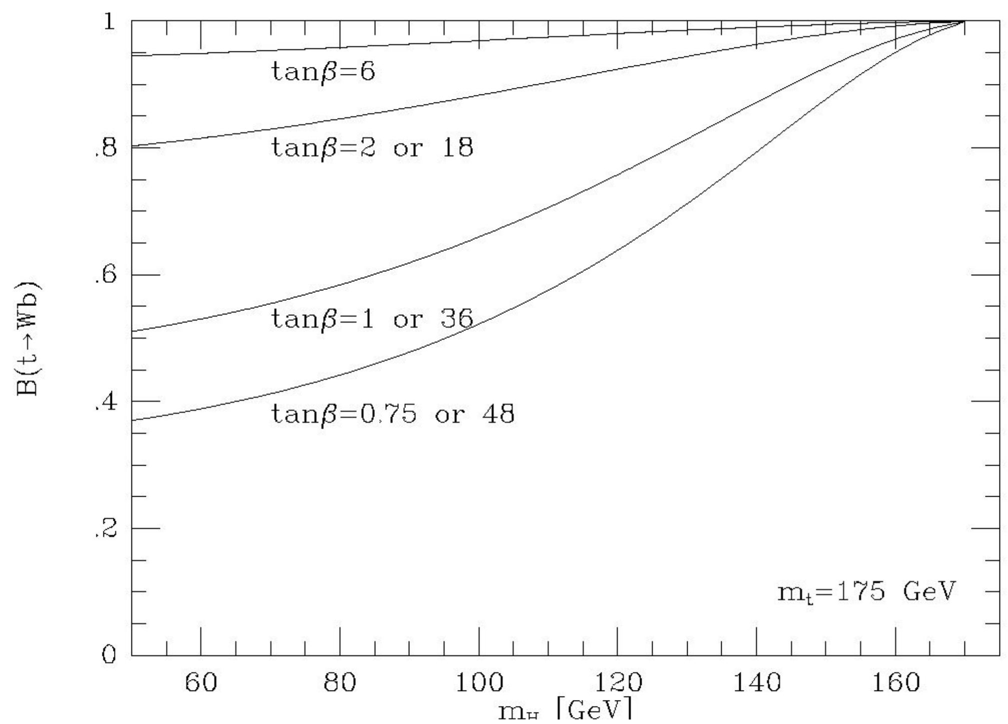
# Bonus: “Unconventional decays”

- MSSM: $t \rightarrow H^{+}b$
- Partial width

$$\Gamma(t\rightarrow Hb)=\frac{G_F m_t^3}{8\pi\sqrt{2}}|V_{tb}|^2 J\left(\frac{M_H}{m_t},\frac{m_b}{m_t},tan\beta\right)$$

- Has a minimum for

$\tan\beta = \mathrm{sqrt}(m_t/m_b) \sim 6$



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